Section 5.6: Substitution

Big picture: Sometimes we can simplify an integral by replacing one independent variable by another independent variable. Look to make things easy for yourself!

1. Our author says that the technique of substitution is really the chain rule backwards.

   Problem 1: Write the derivative of the function $h(x) = f(g(x))$

2. Now suppose that you're looking at the integral

   $$\int f'(g(x))g'(x)dx$$

   Problem 2: What's an anti-derivative?

3. **Problem 3**: Relate the computation of

   $$\int cos(x^2)2xdx$$

   to Part 2 above. Who's playing what role (f, g, h)?

4. One thing that our author focuses on in this section is that **differential**, generally dx, which we find in every integral (e.g. $\int f(x)dx$). He states (to my grand dismay) that "We treat differentials as symbols that do not represent actual quantities...."

Problem 4: From our discussions so far, why do I disagree with this statement? How should we think of the differential dx in $\int f(x)dx$?

5. The key "substitution" that our technique refers to is related to g (in Part 2). You'll notice that recognizing the "f" part of the integrand doesn't really participate in the simplification of the integral (although it does relate to solving it!).

   Now there's one other key feature of substitution that we must consider. So far we haven't really "substituted": we've just recognized the chain rule backwards. The key thing that substitution refers to is $g(x)$: we're going to replace that with a new variable, usually u.

   When we do that, we have that

   $$u = g(x) \qquad \text{and} \qquad du = \frac{du}{dx}dx = \frac{dg}{dx}dx = g'(x)dx$$

   Once we make **these** substitution (of both u and du), the independent variable x is no longer the variable of interest: it's replaced by the new independent variable u. This has one important consequence:

   The limits of integration must be expressed in terms of u, rather than x

   So the limits a and b of an integral undergoing u-substitution,

   $$\int_a^b f'(g(x))g'(x)dx$$

   are changed as well, to

   $$\int_{g(a)}^{g(b)} f'(u)du$$

   Problem 5: Use substitution to compute the integral

   $$\int_0^{-\pi} \cos^2(x)\sin(x)dx$$