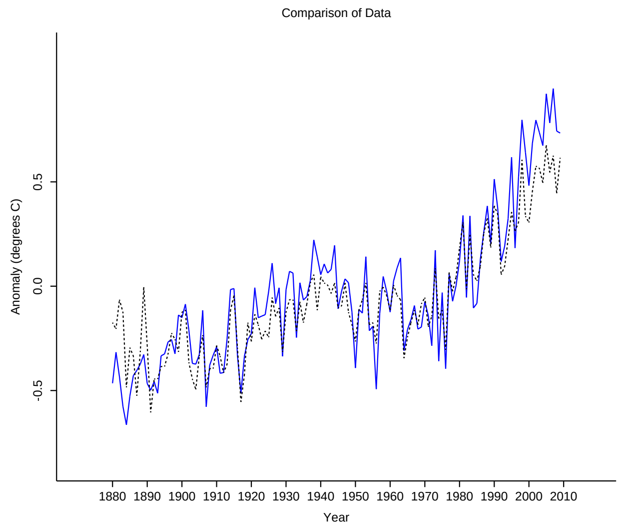
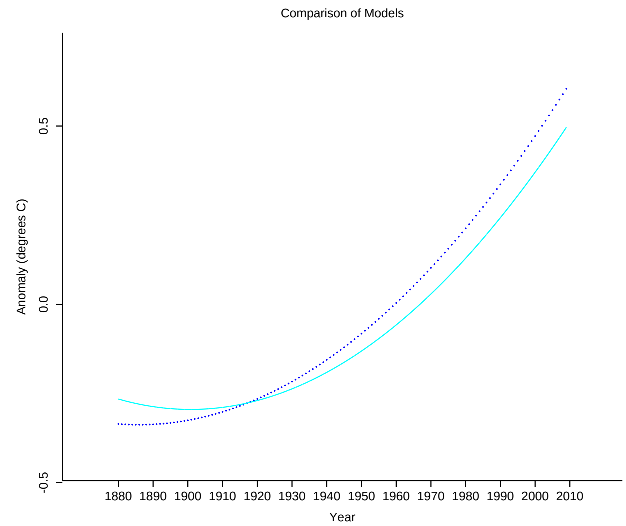
Section 6.1: Area Between Two Curves

The following data is climate data from NOAA (the National Oceanic and Atmospheric Administration): http://www.ncdc.noaa.gov/bams-state-of-the-climate/2009-time-series

One data set is from NOAA, and the other from NASA. Different strategies and different algorithms lead to different results, so, even though these data purport to measure the same thing, they give different pictures. The figures represent a comparison of the data sets, as well as a comparison of models (quadratic models).





Problem 1: Look for a moment only at the data: if you had been following either data set for years and years, since 1880, is there a point at which you would have become alarmed? If so, when?

Problem 2: Ask yourself:

- Is either set of data increasing?
- Would a linear model be a good fit to either?

If a linear fit fails, then we move to more complicated models. The next more complicated polynomial is a quadratic. Each data set has been fit by a quadratic model, determined by a process called **linear regression**, which is informed by calculus:

NASA's data: R Squared:		0.738391	
Linear Regression:	Estimate	SE	Prob
Constant	-0.265678	(3.705582E-2)	0.00000
Linear	-2.807195E-3	(1.327420E-3)	0.03640
Quadratic	6.755786E-5	(9.958556E-6)	0.00000

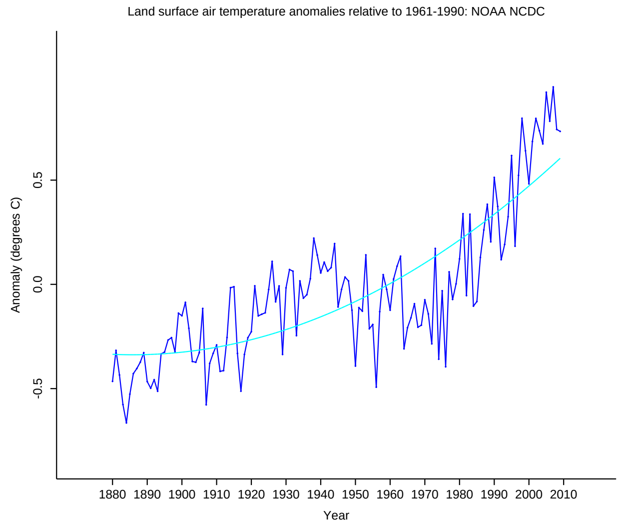
NOAA's data: R Squared:		0.662942	
Linear Regression:	Estimate	SE	Prob
Constant	-0.335557	(5.318269E-2)	0.00000
Linear	-7.330682E-4	(1.905120E-3)	0.70104
Quadratic	6.218198E-5	(1.429257E-5)	0.00003

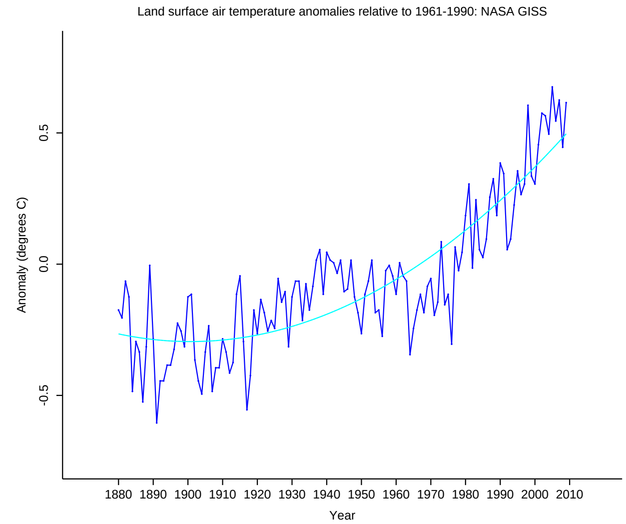
The coefficients of the quadratics are given as "estimates" for the terms multiplying the years (shifted by 1880): hence each model is of the form

$$Q(x) = quadratic * (x - 1880)^2 + linear * (x - 1880) + constant$$

Problem 3: At what point do these two curves intersect (find it exactly, using the quadratics). Will there be another?

Problem 4: Project either of these data sets trends into the future. What do you think land temperatures will be in 2030? Now: what do the quadratics predict?





Problem 5: Let $Q_1(x)$ be the model for the NASA data, and $Q_2(x)$ be the model for the NOAA data. Now suppose that you're looking at the integral

$$\int_I^t (Q_1(x) - Q_2(x))dx$$

where I represents the point of intersection of the two curves, and t represents a time (perhaps the present). What does it represent? Compute it.

Problem 6: Comment on these models. Do you think that they've captured something important? Are they missing something? How could we do better?