Section 7.1: Exponential Functions

In our last lab, we saw that the atmospheric mass density function ρ could be modelled by

$$\rho(x) = 10^{-3(1+x/50)}$$

This is an example of an exponential function (although it may look kind of strange as given above). Let's re-express it, using properties of exponentials:

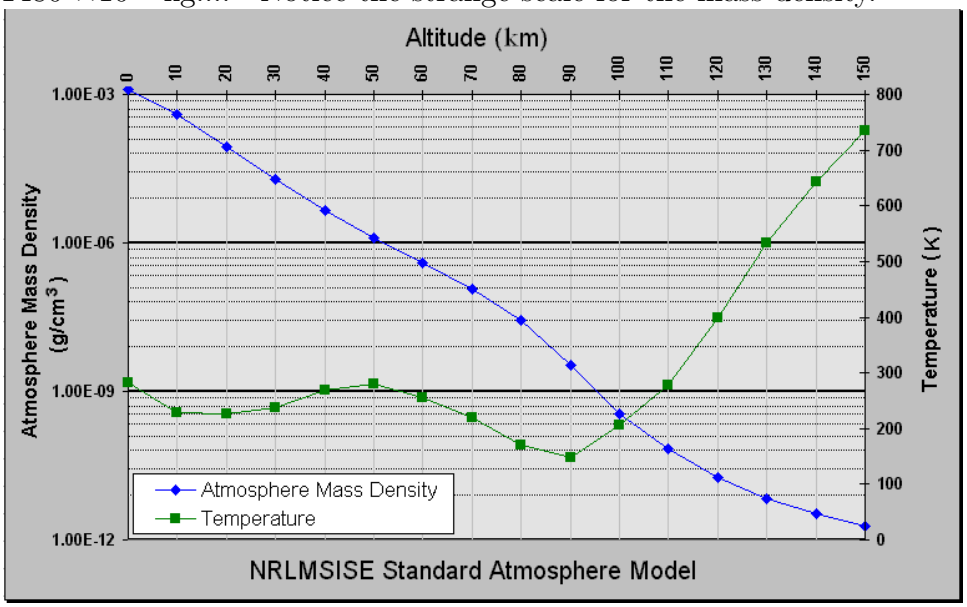
$$\rho(x) = 10^{-3(1+x/50)} = 10^{-3-3x/50} = 10^{-3}10^{-3x/50}$$

Now 10^{-3} is just an ugly constant; and

$$\rho(x) = c\,(10^x)^{-3/50}$$

So we have $\rho(x)$ as some composition of the function 10^x.

Figure 1: `http://upload.wikimedia.org/wikipedia/commons/d/de/Atmosphere_model.png`; According to the National Center for Atmospheric Research, "The total mean mass of the atmosphere is 5.1480 $\times 10^{18}$ kg...." Notice the strange scale for the mass density.



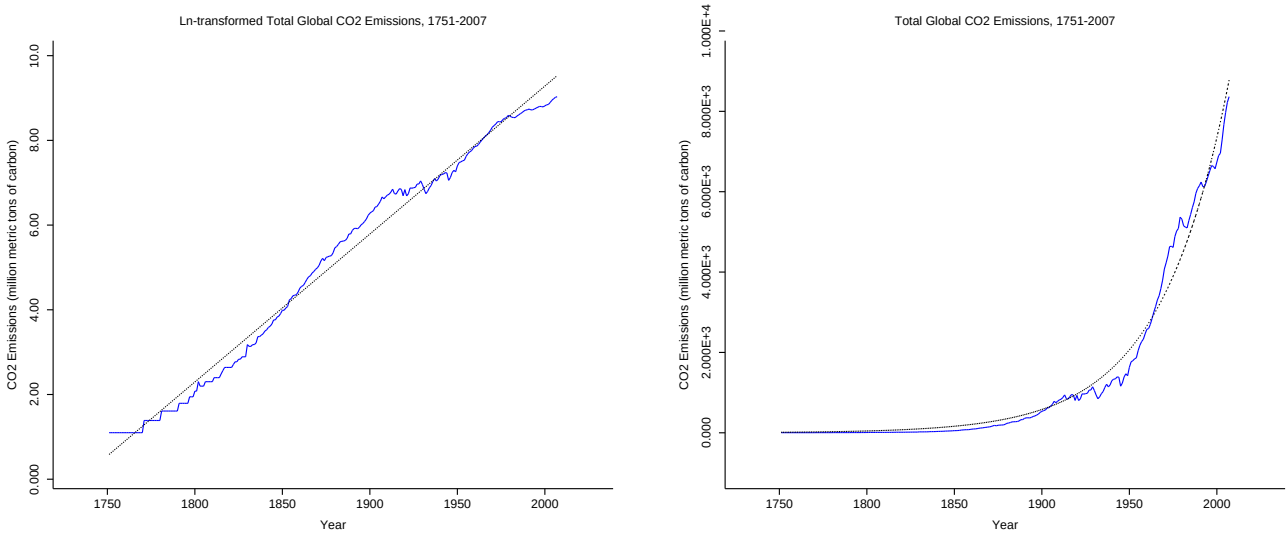
I provided the model – that didn't come with the paper. And the reason I chose this model was because, with the y-axis changing in that funny way (by powers of 10), the graph of the mass density looked linear. [Remember that mathematicians love linear models – they're the simplest really useful models.] If the y-axis is changing by powers, and the graph looks linear, then the phenomenon that y represents is actually exponential (this is a trick that you might choose to learn, if you're a diligent student!).

Now: when you are looking at data that grows superlinearly (i.e. you know that the linear model won't work), you have options. You might consider higher powers (e.g. the quadratic that we looked at in lab08). Or, if you suspect that

- **the rate of change is proportional to the function value,**

then you might consider an exponential function. This is the fundamental physical reason for choosing an exponential model.

Figure 2: From `http://cdiac.esd.ornl.gov/ftp/ndp030/global.1751_2007.ems` – now a dead link. You can get my copy here.. This data concerns Global CO2 Emissions from Fossil-Fuel Burning, Cement Manufacture, and Gas Flaring: 1751-2007. At left is the transformed data, with a linear fit. At right is the original data, and an exponential Emissions model given by $E(t) = e^{8.9+0.025(t-2000)}$, derived from non-linear regression.



1. Criticize the model.

2. Support the model.

3. How might you improve the model?