Global Mean Temperature via Newton's Law

A variant of Newton's law of cooling serves as a simple model for global mean temperature rise (or fall) to time-varying climate forcings. We can imagine that the Earth's atmosphere is the body, and there is heating going on – so the atmosphere will equilibrate to a higher temperature (or even a lower temperature, should we ever manage to initiate any global cooling – a prospect of the new science of "geoengineering").

Here's the model (as presented in an article by Frame and Allen in the rather bleakly titled book "Global Catastrophic Risks"):

$$c_{eff}\frac{dT}{dt} = F(t) - \lambda T$$

where

- T is the global temperature anomaly.
- $F(t)$ is the forcing function, the perturbation to the average energy received by the earth (in W/m^2), which drives the temperature anomaly.
- $c_{eff} > 0$ is the effective heat capacity of the system (governed mainly by the ocean); c_{eff} is like inertia – it represents the sluggishness of the Earth's atmospheric temperature to these forcings.
- $\lambda > 0$ is a feedback parameter.

In the absence of $F(t)$, anomaly should (and would) go to zero.

This model is effectively the same as the model for Newton's law of cooling when $F(t)$ is constant. To make the model more realistic, $F(t)$ is not constant, but rather reflects the various sources or sinks of energy input into the Earth from various time-varying phenomena (e.g. rising CO2, methane releases, solar flares, jet contrails, changes in Earth's albedo – reflectivity, etc.).

1. If energy forcing is zero – that is, $F(t) = 0$ – the model becomes

   $$c_{eff}\frac{dT}{dt} = -\lambda T$$

   Solve this separable differential equation for T, assuming that $T_0 = .5^oC$. How do the parameters λ and c_{eff} affect the solution? [Hint: This can be solved using the general method for solving differential equations, i.e. staring at them until the solution comes to you.]

2. If energy forcing increases as the anomaly increases – e.g. methane defrosting under warming seas, say, or plants dying and so taking less CO_2 from the air – we might see that $F(t)$ is proportional to $T(t)$ – say $F(t) = \alpha T$. In that case, the model would become

$$c_{eff}\frac{dT}{dt} = \alpha T - \lambda T = (\alpha - \lambda)T$$

where we might assume that $\alpha - \lambda > 0$. Solve this separable differential equation for T, assuming that $T_0 = .5^o C$.

This, too, can be solved using the general method.

3. Alternatively, the feedback parameter λ may be temperature varying: that is, as temperature anomaly rises or falls, the ability of the system to restore itself to equilibrium may change (either getting faster or more sluggish).

$$c_{eff}\frac{dT}{dt} = F(t) - \lambda(T)T$$

An example of this sort of behaviour is where increasing temperature causes a change in ocean circulation, which reduces the ability of the Earth to distribute temperature between the ocean and atmosphere. Anomaly will increase ($\frac{dT}{dt} > 0$) so long as $F(t) - \lambda(T)T > 0$.

Do we want $\lambda(T)$ to grow with increasing T, or decrease? Explain.