Homework 10

MAT 227, Spring 2009

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11. Write $\int \sin(2x-4)\,dx$ in term of $u = 2x-4$, then evaluate the integral.

If $u = 2x-4$, then $du = 2\,dx$ or $dx = \frac{1}{2}\,du$. The integral $\int \sin(2x-4)\,dx$ becomes

$$\begin{aligned}
&\int \sin(u)\,\tfrac{1}{2}\,du \\
&= \tfrac{1}{2}\int \sin(u)\,du \\
&= \tfrac{1}{2}(-\cos(u)) + C \\
&= -\tfrac{1}{2}\cos(2x-4) + C
\end{aligned}$$

57.What are the new limits of integration if we apply the substitution $u = 3x+\pi$ to the integral $\int_0^\pi \sin(3x+\pi)\,dx$?

If $u = 3x+\pi$, then $du = 3\,dx$ with $x = 0 \Rightarrow u = \pi$ and $x = \pi \Rightarrow u = 4\pi$. The new integral would be

$$\int_0^\pi \sin(3x+\pi)\,dx = \int_\pi^{4\pi} \tfrac{1}{3}\sin(u)\,du$$

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62. Use the change of variables formula to evaluate $\int_{-1}^{2} \sqrt{5x+6}\,dx$.

Use the substitution $u = 5x+6$

$$\Longrightarrow du = 5\,dx \text{ or } dx = \tfrac{1}{5}\,du$$

and

$$x = -1 \Rightarrow u = 1$$
$$x = 2 \Rightarrow u = 16$$

then

$$\begin{aligned}
&\int_{-1}^{2} \sqrt{5x+6}\,dx = \int_1^{16} \sqrt{u}\,\tfrac{1}{5}\,du \\
&= \tfrac{1}{5}\int_1^{16} u^{1/2}\,du \\
&= \tfrac{1}{5}\,\tfrac{u^{3/2}}{3/2}\Big|_1^{16} \\
&= \tfrac{2}{15}\left(16^{3/2} - 1^{3/2}\right) = \tfrac{126}{15} = \tfrac{42}{5} = 8.4
\end{aligned}$$

70. Use the change of variables formula to evaluate $\int_0^{\pi/4} \tan^2(x) \sec^2(x)\, dx$.

Use the substitution $u = \tan(x)$

$$\Longrightarrow du = \sec^2(x)\, dx$$

and

$$x = 0 \Rightarrow u = \tan(0) = 0$$
$$x = \pi/4 \Rightarrow u = \tan(\pi/4) = 1$$

then

$$\int_0^{\pi/4} \tan^2(x) \sec^2(x)\, dx = \int_0^1 u^2\, du$$
$$= \frac{u^3}{3} \Big|_0^1$$
$$= \frac{1}{3}$$