Homework 9

MAT 227, Spring 2009

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4. Calculate du for $u = 2x^4 + 8x$.

Since $du = u'(x)\,dx$, $du = \left(8x^3 + 8\right)dx$.

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10. Write $\int x(x+1)^9\,dx$ in terms of u and du where $u = x + 1$. Then evaluate the integral.

If $u = x + 1$, then $du = dx$ and also $x = u - 1$. So

$$\begin{aligned}
&\int x(x+1)^9\,dx \\
&\quad = \int (u-1)\,u^9\,du \\
&\quad = \int u^{10} - u^9\,du \\
&\quad = \frac{u^{11}}{11} - \frac{u^{10}}{10} + C \\
&\quad = \frac{(x+1)^{11}}{11} - \frac{(x+1)^{10}}{10} + C
\end{aligned}$$

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16. Write $\int x\sqrt{4x-1}\,dx$ in terms of u and du where $u = 4x - 1$. Then evaluate the integral.

If $u = 4x - 1$, then $du = 4\,dx$. This means $dx = \frac{1}{4}\,du$ and $x = \frac{u-1}{4}$. So

$$\begin{aligned}
&\int x\sqrt{4x-1}\,dx \\
&\quad = \int \frac{u-1}{4}\sqrt{u}\,\frac{1}{4}\,du \\
&\quad = \frac{1}{16}\int (u-1)\,u^{1/2}\,du \\
&\quad = \frac{1}{16}\int \left(u^{3/2} - u^{1/2}\right)du \\
&\quad = \frac{1}{16}\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right) + C \\
&\quad = \frac{1}{16}\left(\frac{2}{5}\left((4x-1)^{5/2} - \frac{2}{3}(4x-1)^{3/2}\right) + C = \frac{1}{40}(4x-1)^{5/2} - \frac{1}{24}(4x-1)^{3/2} + C\right.
\end{aligned}$$

26. Evaluate $\int x^2\left(x^3+1\right)dx$.

Rearrange $\int x^2\left(x^3+1\right)dx$ as $\int\left(x^3+1\right)x^2\,dx$. Try the substitution $u = x^3 + 1$, then $du = 3\,x^2\,dx$ or $x^2\,dx = \frac{1}{3}\,du$.

$$\begin{aligned}\int x^2\left(x^3+1\right)dx &= \int \frac{u}{3}\,du \\ &= \frac{u^2}{6} + C \\ &= \frac{\left(x^3+1\right)^2}{6} + C\end{aligned}$$

30. Evaluate $\int (2x+1)\left(x^2+x\right)^3\,dx$

Rearrange $\int (2\,x+1)\left(x^2+x\right)^3\,dx$ as $\int\left(x^2+x\right)^3(2\,x+1)\,dx$. Try the substitution $u = x^2 + x$, then $du = (2\,x+1)\,dx$.

$$\begin{aligned}\int (2\,x+1)\left(x^2+x\right)^3\,dx &= \int u^3\,du \\ &= \frac{u^4}{4} + C \\ &= \frac{\left(x^2+x\right)^4}{4} + C\end{aligned}$$