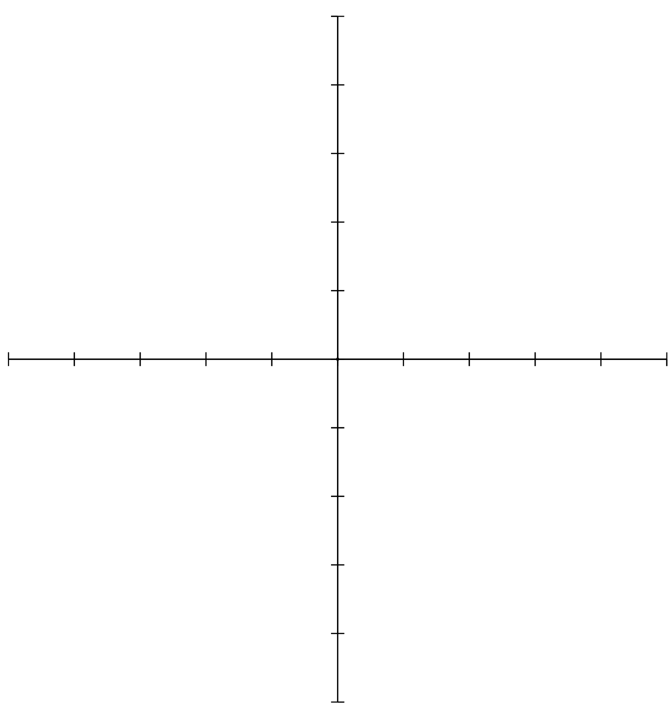
MAT227 Final (Fall 2010)

Name:

Directions: Problems are not equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). You are allowed to skip **five** five-point problems (all sub-parts of a problem are five points, unless otherwise specified. Write skip clearly on parts skipped.
Good luck!
Problem 1: (25 pts) Each of the following five problems is worth five points.

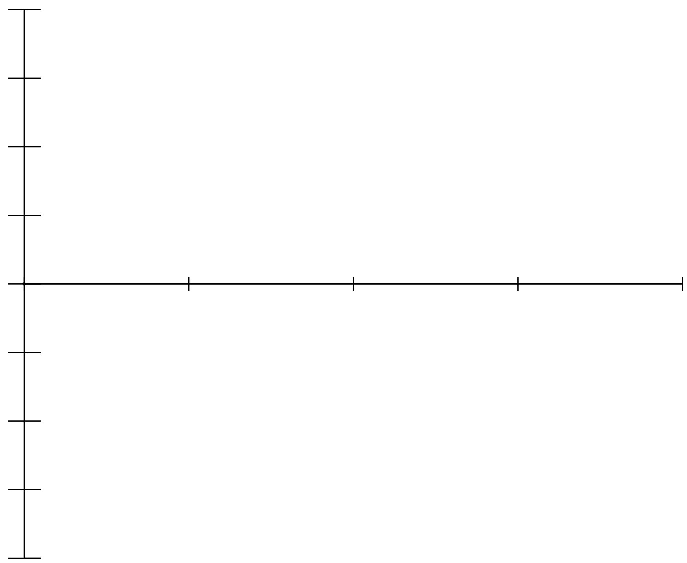
a. Evaluate $\lim_{x \to 0} \frac{\sin^2(x)}{x^2}$.

b. Graph the function $f(x) = -1 + \frac{3}{x-1}$ on the interval [-2,4], and then its inverse.



c. The following three parts all pertain to $f(x) = \sin(x)$:

i. (5 pts) Rotate the graph of f about the x-axis over the interval $[0, \pi]$. Compute the volume of the resultant solid.

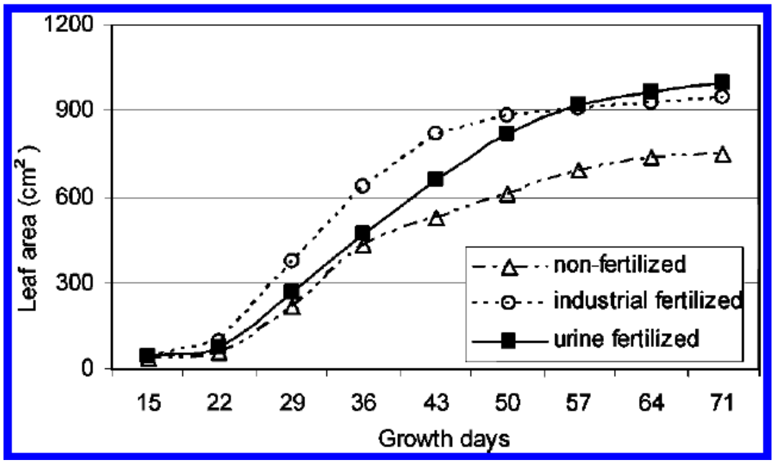


ii. (5 pts) Compute the arc length of f on the interval $[0, \pi]$.

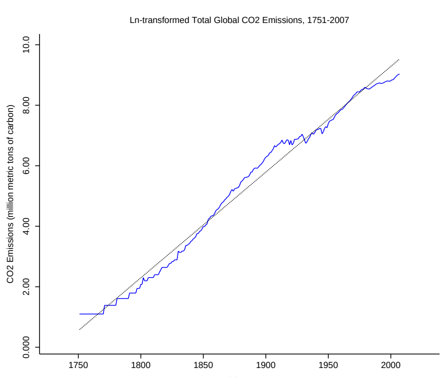
iii. (5 pts) Compute the surface area of the rotated solid above.

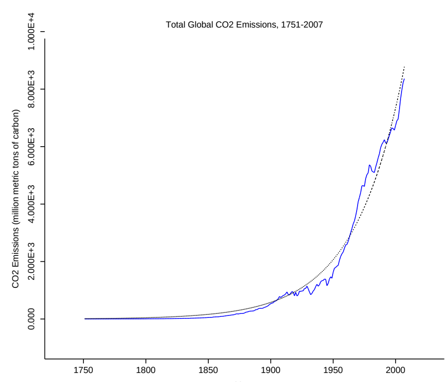
Problem 2: (25 pts) The following problems refers to the labs (each is worth five points).

a. In the first lab, we considered the value of urine fertilization to cabbage production. Relate the energy provided to a plant by a leaf to an integral using the information in the following figure, and explain which treatment is best (provides the most energy to the plant).



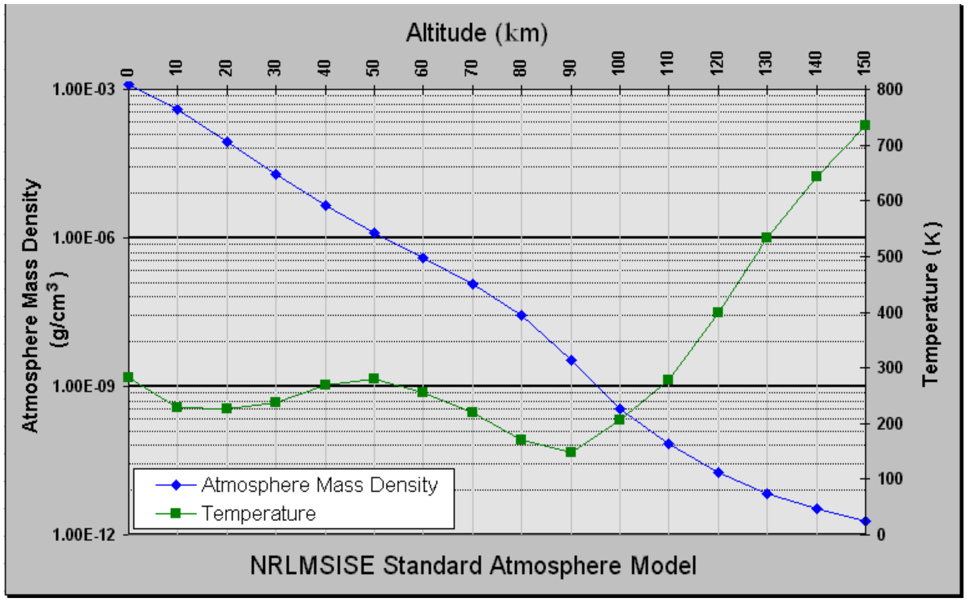
b. What does the graph on the left have to do with modeling Total CO_2 emissions by an exponential function?





c. "The pipe's only six inches in diameter! How could we possibly get 20,000 barrels of oil out of it per day?!" Explain what the lab was about, and give some indication of how we were able to answer that question.

d. How could one use the data in the following image to compute the mass of the atmosphere?

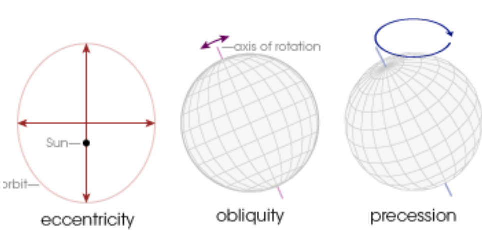


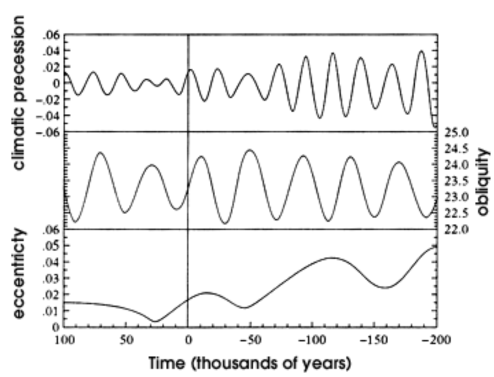
e. What do the following two graphs have to do with

i. each other?

ii. parametric curves?

iii. our climate?





Problem 3: (25 pts) Consider $I = \int_0^{\pi/2} x \cdot \sin(x)dx$:

a. (10 pts) Integrate by parts. Show all steps in your integration.

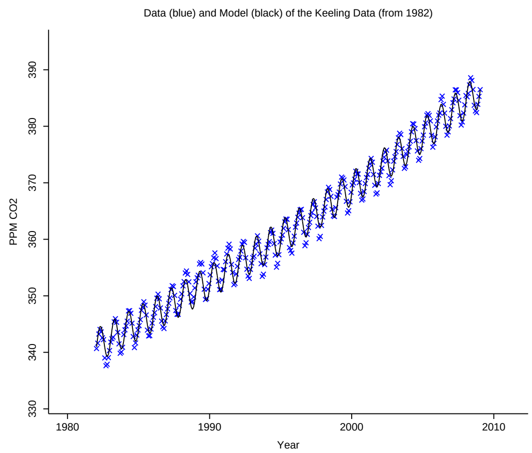
b. (10 pts) Approximate I using the left and right rectangle rules with four subintervals, L_4 and R_4; midpoint rule M_4; the trapezoidal rule T_4. Work below, and summarize in the table.

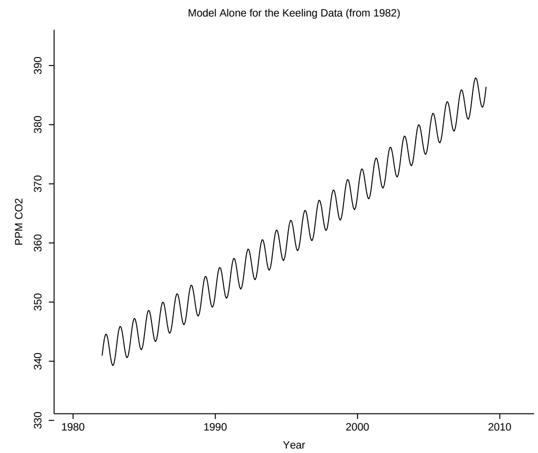
Method	estimate	error (true-estimate)
L_4		
R_4		
M_4		
T_4		

c. (5 pts) Compare the errors made using each approximation method. In each case explain why the error makes sense.

Problem 4: (10 pts) The primary forcing for global increase in temperature is CO_2, which is

increasing world-wide. The data on that was first compiled and brought to the world's attention by Charles Keeling, in Hawaii. The model is a quadratic with a sinusoidal piece added on, and t is





in years from January, 1982:

$$PPM(t) = 341.274 + 1.288t + .0143t^2 - 3.02387\sin(6.28641(t + 0.482795))$$

a. What is the rate of change of PPM, according to our model?

b. What is the total change in PPM from January, 1992 until January, 2002, according to our model? How can one write it as an integral using part a.?

Problem 5: (20 pts) Some shorter problems.

a. Use the limit definition of the derivative (and L'Hopital) to demonstrate that $(e^x)' = e^x$.

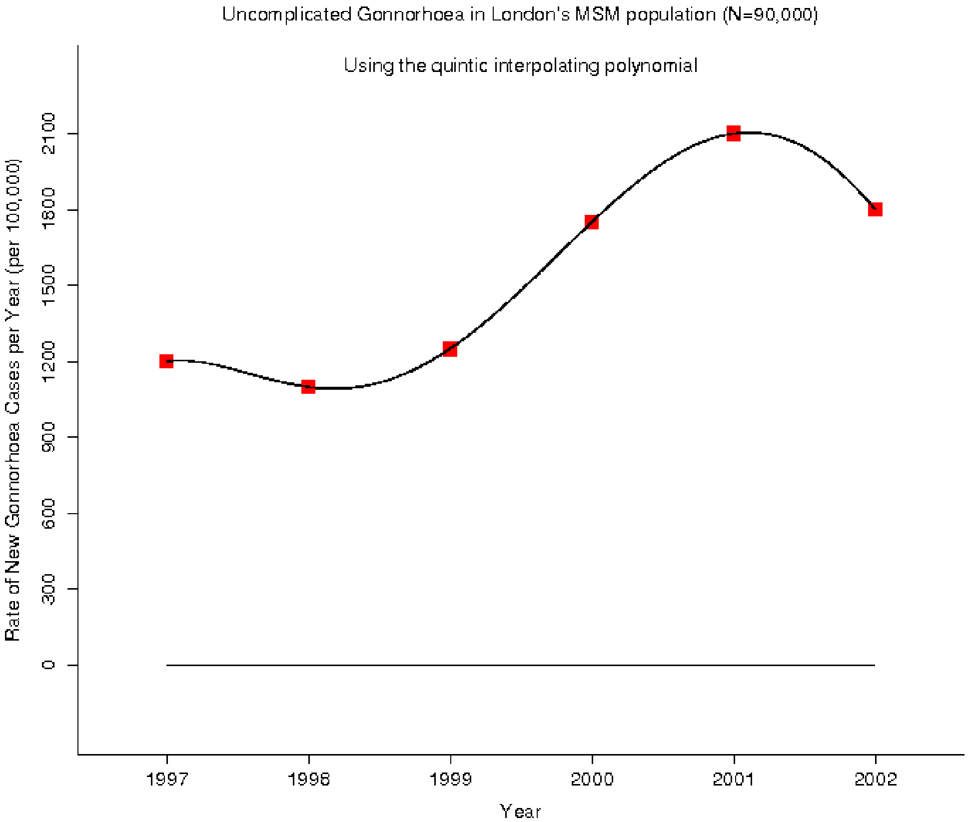
b. Express $f(x) = \log_3(x)$ as an equivalent function, replacing $\log_3$ with ln. (Show steps!)

c. Explain why $\sin^{-1}(\sin(2\pi/3) = \pi/3$ (a graph would be wise).

d. Evaluate by substitution: $\displaystyle\int \frac{\cos(2x)dx}{(1+\sin(2x))^2}$. Show all steps.

Problem 6: (15 pts) Several times we derived the formula for arc length L in class. It was relatively straightforward, using only discretization and the Pythagorean formula.

a. (5 pts) Show how to derive it now, using the following curve as a prop:



b. (5 pts) Does arc length have a physical meaning for this graph? If so, what? If not, why not?

c. (5 pts) What is the meaning of $\int_{1997}^{2002} f(t)dt$, where f is the function graphed above? What are its units?

Problem 7: (10 pts) Consider a planet on an elliptical orbit: The Sun is at the origin and the plane of the orbit has coordinates x and y. We can write parametric equations for the time t, and coordinates x and y, in terms of an independent variable ψ:

$$t = \frac{T}{2\pi}\left(\psi - \varepsilon \sin \psi\right) \tag{1}$$

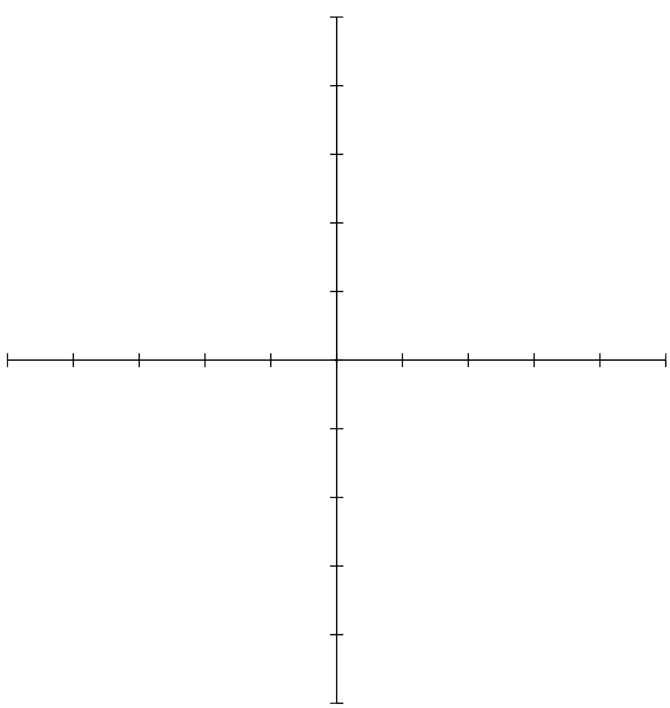
$$x = a\left(\cos \psi - \varepsilon\right) \tag{2}$$

$$y = a\sqrt{1-\varepsilon^2}\,\sin \psi \tag{3}$$

$T =$ period of revolution, $a =$ semimajor axis, and $\varepsilon =$ eccentricity. The eccentricity of the Earth's orbit is currently about 0.01671022. Its semimajor axis is 1.00000011 AU. T= 1 Earth year.

a. Compute the speed of the planet (as a function of ψ). At what value of ψ would the actual speed of the planet (in time t) be greatest?

b. Graph the orbit of the planet, starting at $\psi = 0$. Show the direction of travel, and where the Sun would be, relative to the origin.



Problem 8: (5 pts) Consider the curve given in polar coordinates by $r = 2\cos(3\theta)$. Plot the curve for $\theta \in [0, 2\pi]$, indicating direction of travel and how many times the graph is traversed: