MAT227 Exam 1 (Fall 2010)

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1: (28 pts) Using 6 equally spaced sub-intervals, approximate the integral

$$I = \int_0^4 \frac{x}{\sqrt{x^2+9}} dx$$

using right endpoint, left endpoint, and trapezoidal methods. For R_6, show all details. You may use your calculator for the others.

a. (8 pts) $R_6 =$

b. (4 pts) $L_6 =$

c. (4 pts) $T_6 =$

d. (4 pts) Compare the exact value of the integral to the approximations above (you may use your calculator, if necessary, to calculate the exact value). Which method gives the best approximation? Are you surprised?

e. (8 pts) (Problem 1, cont.)

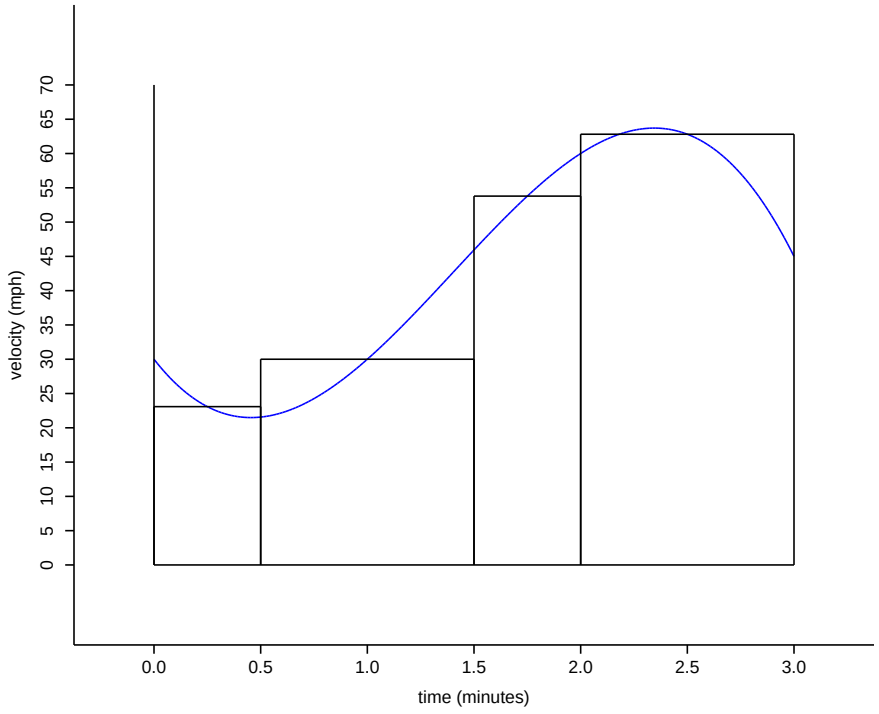
This integral can be calculated exactly using substitution. Identify the appropriate substitution $u = g(x)$, and carry out the integration (show all steps).

Problem 2: (10 pts) Consider the function $f(x) = \dfrac{x+1}{\sqrt{x}}$: Write an antiderivative F of f such that $F(1) = 2$ in two different ways:

a. Using an integral representation, and

b. By finding an explicit function $F(x)$ using the FTC Part I.

Problem 3: (14 pts) The continuous graph in this figure represents the velocity of an object over a 3-minute span, in mph.

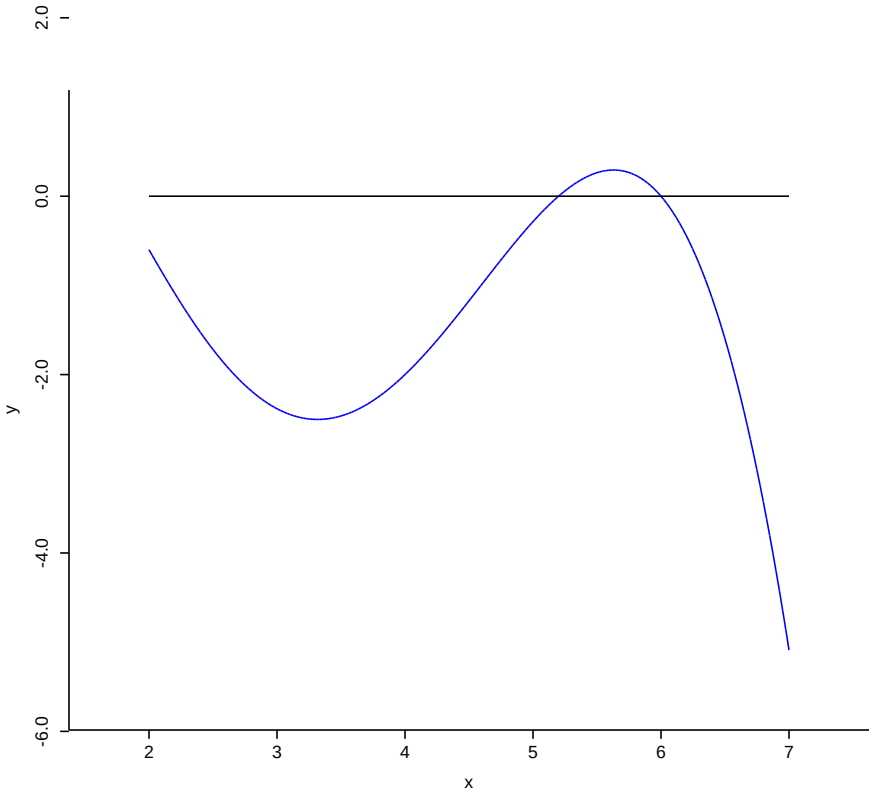


a. (8 pts) Use the Riemann sum represented in the figure to approximate the distance traveled. Pay attention to units.

b. (4 pts) Identify the partition used and the set of intermediate points used in the Riemann sum.

c. (2 pts) What is the name of the approximation method illustrated in this figure?

Problem 4: (16 pts) Let $A(x) = \int_0^x f(t)dt$, with the graph of f represented in the following figure:



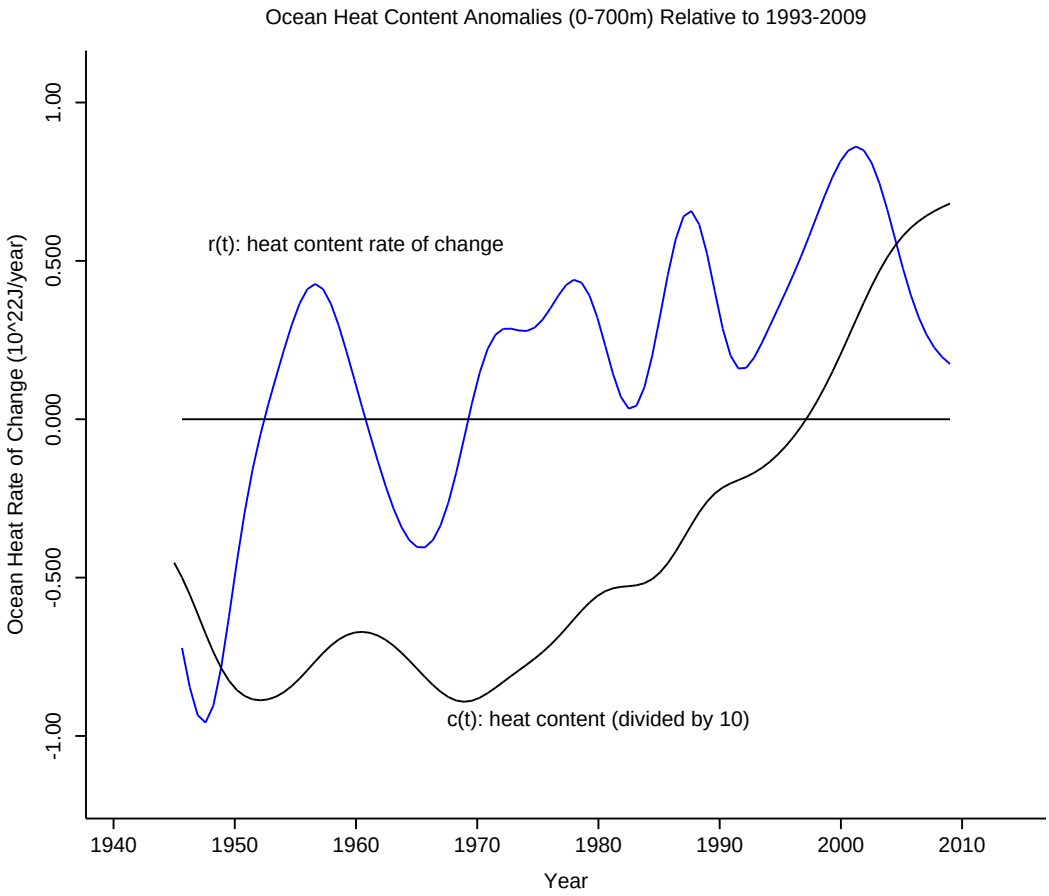
a. Where does $A(x)$ have a local minimum?

b. Where does $A(x)$ have a local maximum?

c. True or false (with reasons!): $A(x) < 0$ for all x in the interval shown.

d. Write a formula for $\dfrac{d}{dx}\int_0^{x^2} f(t)dt$ in terms of A, f, and x.

Problem 5: (12 pts) This figure shows the graph of ocean heat content rates of change $r(t)$ over the past 70 years or so. It synthesizes and summarizes 7 data sets obtained from NOAA. Also included is the estimated heat content ($c(t)$), although I had to rescale its graph to fit it on this plot (I divided its values by 10). Answer the following questions in the space below:



a. What does the area under the graph of $r(t)$ over the interval [1970,2010] represent?

b. How would you describe the trend in $c(t)$ since 1970?

c. What aspects of $r(t)$ inform the trend in $c(t)$?