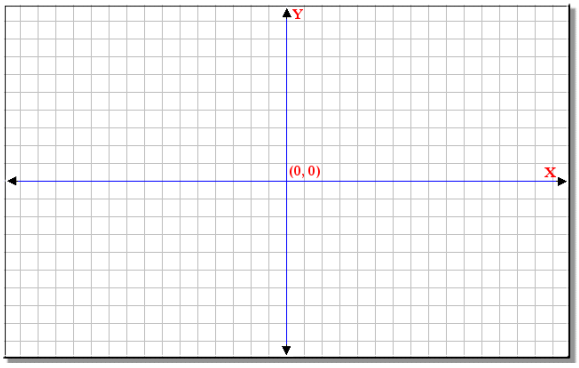
MAT227 Test 2 (Fall 2010): Integrals, Exponentials, and Inverses

Name:

Directions: Problems are not equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1: (10 pts) Consider the functions $f(x) = x^2$ and $g(x) = x^{1/3}$, defined on [-1,1].

a. (2 pts) Draw the two functions carefully over this interval, indicating which is which, and shade in the physical area between the two curves.



b. (2 pts) First estimate the area between the two curves. Explain your estimate.

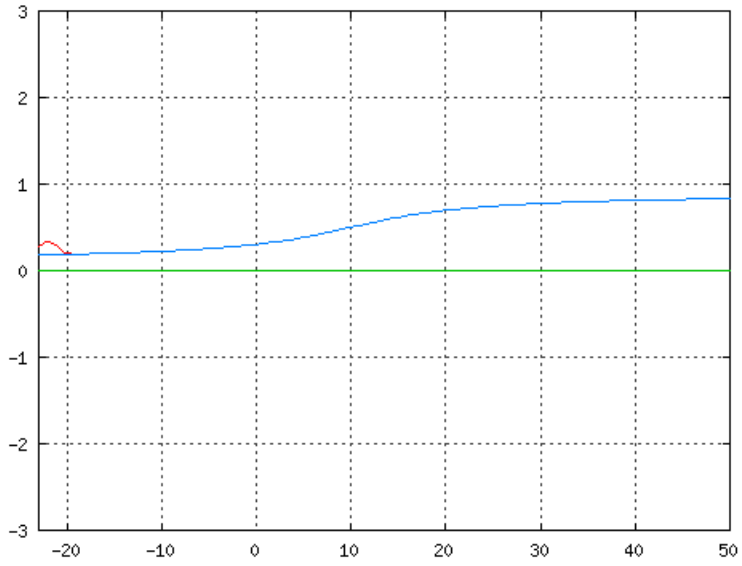
c. (3 pts) Write down the integral that represents the area between the two curves.

d. (3 pts) Calculate the area between the two curves (you may use your calculator).

Problem 2: (10 pts) A baseball bat will be created on a lathe at the Louisville Slugger plant by rotating the curve of the function f (given in inches)

$$f(x) = .15 * e^{\left(\frac{-(x+22)^2}{2}\right)} + 0.5 + .25 * tan^{-1}\left(\frac{x-10}{10}\right)$$

about the x-axis, on the interval [-23,50] (cm). Here's the design on the company printer:



a. (7 pts) Draw the bat, then set up (but do not solve) the integral for the volume of the bat. What are the units of your answer?

b. (3 pts) The function f is a sum of three distinct functions. Identify the role of each piece of f in the design of this baseball bat. [Why did I include each term? Each term is there for a reason!]

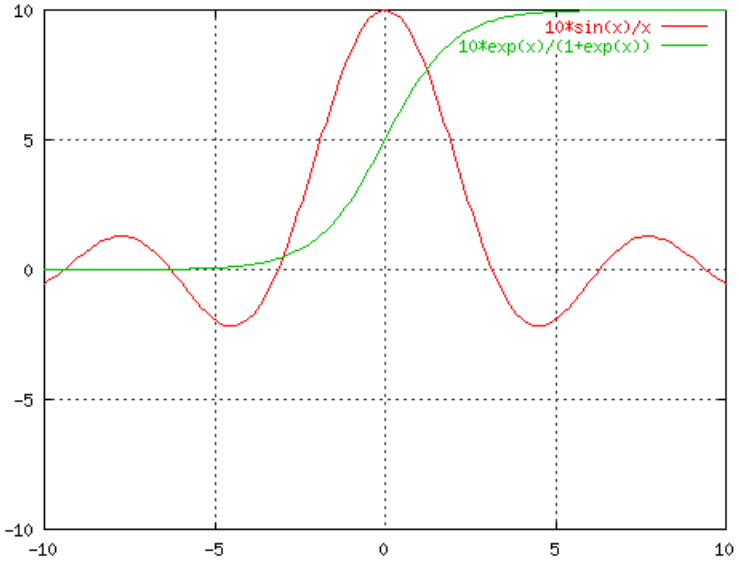
Problem 3: (20 pts) Variety pack – show work!

a. Re-express the function $f(x) = 2^x$ as an exponential function with base e.

b. Solve for x: $\ln(x+1) - 2\ln(x) = 1$

c. If $\theta = cos^{-1}(x)$, what is $\tan(\theta)$ expressed as a function of x, without trig or arc-trig functions? (Hint: draw a triangle.)

d. Carefully graph the inverse of the invertible function whose graph is given here. Point out the non-invertible function, explain how you know it's non-invertible, and how you would proceed if asked to provide an inverse.



Problem 4: (20 pts) This figure, found in James Lovelock's 2006 book *The Revenge of Gaia: Earth's Climate Crisis and the Fate of Humanity*, shows global temperature change over time (in ^oC), as well as 1988 predictions for future changes based on various scenarios. Three explicit data points that appear to lie on the graph are (1860,-.5), (1977,0), and (2006, .7).

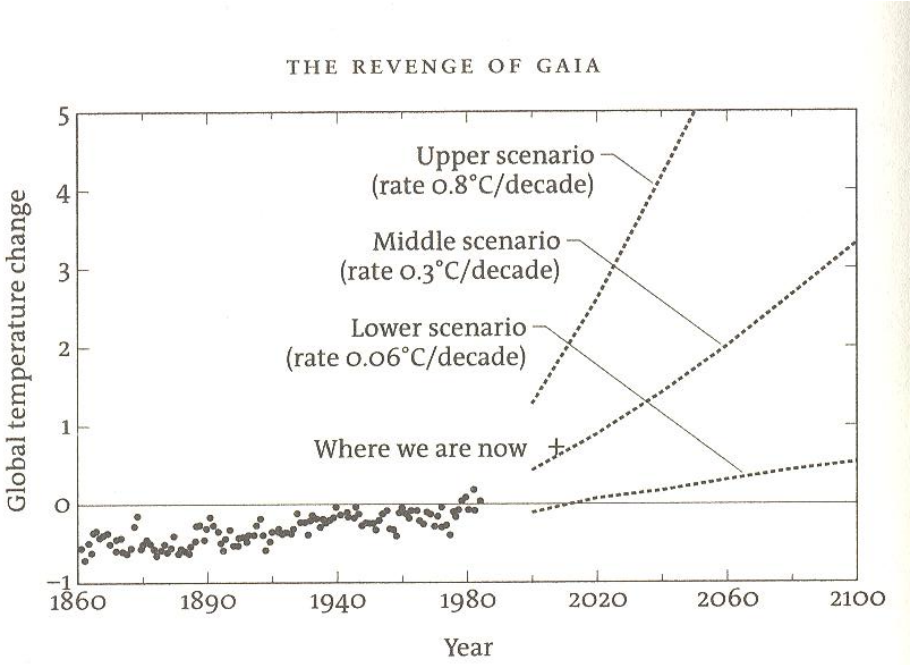


Figure 2. *Climate forecasts made in 1988.*

a. (5 pts) Assuming an exponential model of the form

$$TC(t) = a + be^{c(t-1860)}$$

write three equations using the three data points that would help to determine a, b, and c.

b. (5 pts) Consider the choice of parameters $a = -0.516$, $b = 0.0163$, and $c = 0.0295$. Carefully plot this model over the data in the graph above. How well does the model fit the three explicit data points above?

c. (5 pts) Does the set of all data points appear to be consistent with this exponential model? Justify.

d. (5 pts) In what year does this model predict a global temperature change of 5 degrees?

Problem 5: (10 pts) What's the average value of the function $f(x) = x^2$ on the interval [0,2]?

a. (2 pts) First estimate it, and explain your estimate; then

b. (4 pts) Calculate it exactly.

c. (4 pts) Find the limit

$$\lim_{x \to 0} \frac{1}{x} \int_0^x t^2 dt$$