

MAT115 Test 3 (Spring 2010): Chaos, Infinity, Logic

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1: (30 pts)

a. (10 pts)

i. (2 pts) What is a statement (or proposition)?

ii. (3 pts) What is a premise? What is a conclusion?

iii. (3 pts) Give an example of your own illustrating the three different **types** of statements considered by Lewis Carroll in the logic game.

iv. (2 pts) Write a legitimate negation of the following: "All students are brave."

b. (20 pts) Play Lewis Carroll's Logic Game on three of the following four trios to determine whether the conclusion (third line) is justified or not.

i. No bald creature needs a hairbrush;
No lizards have hair.
No lizard needs a hairbrush.

ii. Some dreams are terrible;
No lambs are terrible.
Some dreams are not lambs.

iii. All ducks waddle;
Nothing that waddles is graceful.
No duck is not graceful.

iv. Spiders spin webs;
Some creatures, that do not spin webs, are savage.
Some spiders are savage.

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Problem 2: (30 pts) Infinity

a. (5 pts) Describe the setup and explain the conclusion of the pingpong ball conundrum.

b. (12 pts) Group the following sets of numbers into two groups, where all of the sets in a group are the same size as infinite sets (have the same cardinality). Indicate which group includes the larger sets.

- i. Real numbers
- ii. Natural numbers
- iii. Even natural numbers
- iv. Fibonacci numbers
- v. Irrational numbers
- vi. Rational numbers

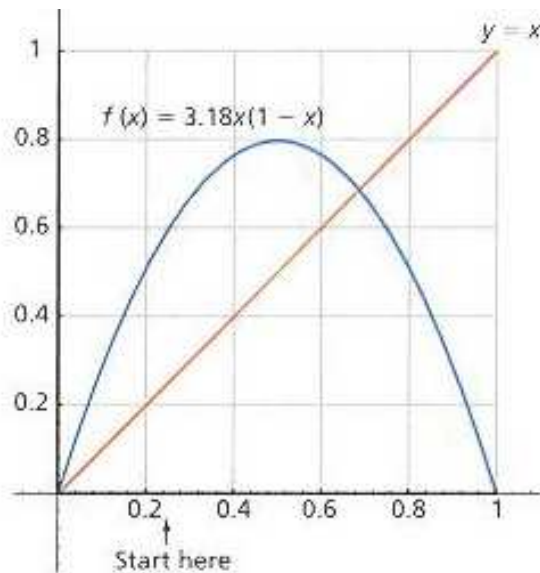
- c. (5 pts) Give a real, useful example of the use of the notion of one-to-one correspondence from your own experience. Why do we need this notion when it comes to comparing sizes of infinite sets?
- d. (8 pts) The Hotel ∞ is full up: each naturally-numbered room has an occupant. The Bus ∞ , with naturally-numbered seats, pulls up. Must we send all that business away? Either find all the bus riders rooms, or explain why that's impossible. Remember: no guest already in the hotel may be asked to move more than once! (Makes 'em cranky....)

Problem 3: (30 pts) Chaos

a. (5 pts) Describe the Lorenz equations, and why they were important in the history of chaos.

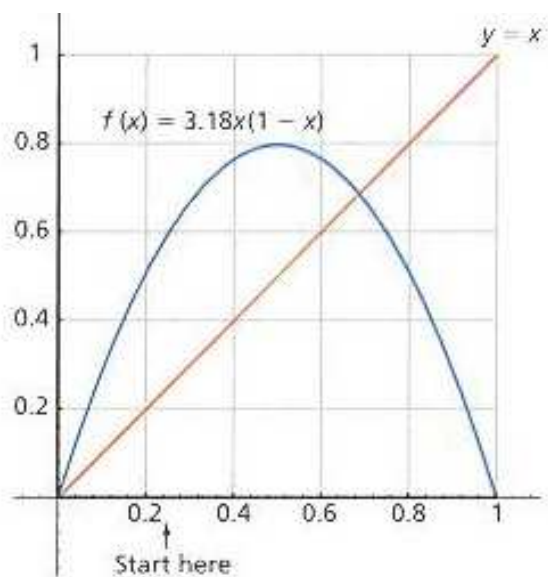
b. (5 pts) How would you explain the notion of chaos to your classmates? What are important elements?

c. (10 pts) In the following figure, start on the horizontal axis where it says “Start here”, and **carefully** draw five iterations of the cobweb plot (that is, find the next five values after the starting value). What conclusions do you draw about the long term behavior of the cobweb path?



d. (10 pts) For this problem, ignore the “Start here” in the figures.

- i. On the figure below indicate two starting points from which you would start and never leave those points.



- ii. On the figure below indicate two points from which you would start and never return.

