MAT129 Test 1 (Spring 2010): Functions, Limits, and Derivatives

Name:

Directions: All problems are equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Note**: you may skip **one** of the following problems. Write skip across it, so I'll easily know which one.

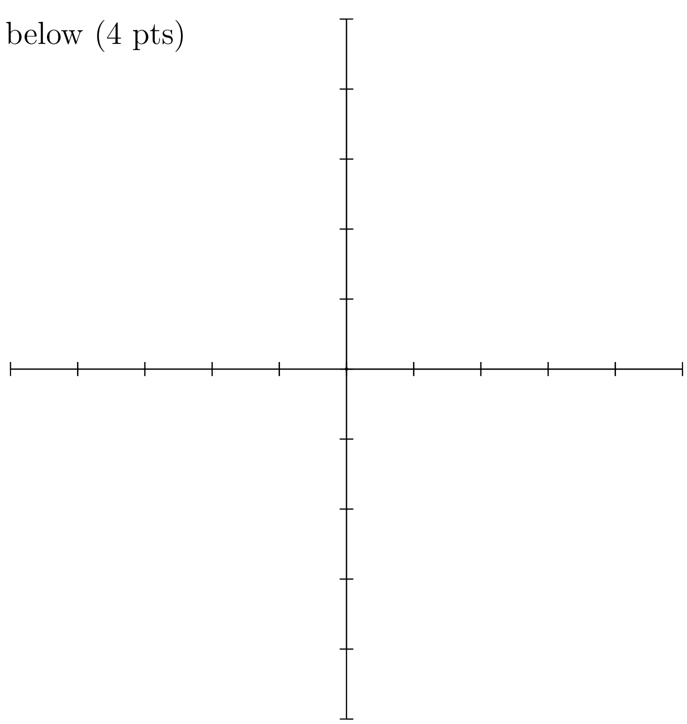
Good luck!

Problem 1. Consider the function f defined by

$$f(x) = \begin{cases} x - b & x < 1 \\ ax + 1 & 1 \leq x < 2 \\ (x-1)^2 + 1 & x \geq 2 \end{cases}$$

a. Choose the parameters a and b so that f becomes a continuous function (6 pts)

b. Carefully graph f below (4 pts)



Problem 2. Use the limit definition to derive the derivative of the function $f(x) = \dfrac{2x}{x+1}$ at $x_0 = 1$. Then use this derivative value to write the equation of the tangent line to the function at $(1, f(1))$.

Problem 3. Describe the following functions that you encounter at the function zoo: for example, you might describe the type of function, the domain of definition, continuity and differentiability properties, the symmetry properties....

a. $f(x) = \dfrac{x}{x^4 - 1}$

b. $g(x) = \sqrt{4 - x^2}$

Problem 4. Find the following limits, citing appropriate limit laws, tricks you've learned, properties of the classes of functions making up f, etc. Carefully explain how you found the limit, justifying each step.

Include some numerical verifications. A graphical or numerical solution is better than nothing, but will only get half credit.

a. $\displaystyle \lim_{x \to -2^+} \frac{x+2}{x-2}$

b. $\displaystyle \lim_{x \to 0} \frac{\sqrt{x+2}}{|x-2|}$

Problem 5. Given the following data: Suppose that we wish to estimate the rate of change of h

t	1	2	3	4	5
h(t)	0	0	2	6	12

with respect to t at $t = 3$.

Make at least two reasonable estimates of the rate of change, using different elements (data values) of the table.

Problem 6. Suppose that f and g are both differentiable at $x = x_0$. Prove that the sum function $f(x) + g(x)$ is differentiable at $x = x_0$, and that the derivative value is given by $f'(x_0) + g'(x_0)$. **Hint**: you might consider using the limit definition of the derivative, and one of my favorite tricks in the book!