MAT129 Test 3 (Spring 2010): Chapter 4, 5.1-5.2

Name:

Directions: Problems are not equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1 (24 pts). This first problem concerns the function $f(x) = \dfrac{-(x^2 - 14x + 40)}{(x-6)^2}$ which we want to study and graph. You are welcome to make good use of your calculator on this problem.

a. (2 pts) What kind of function is this, and what is its domain?

b. (2 pts) Give equations of all asymptotes.

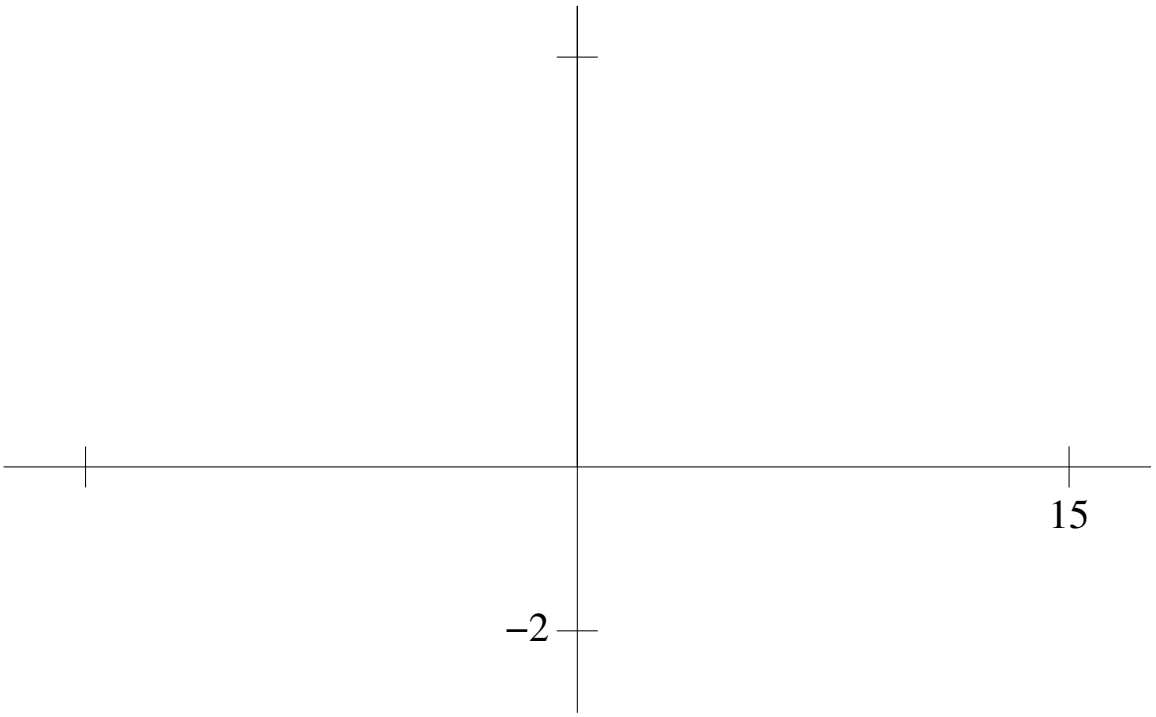
c. (2 pts) What are the roots of the function (where $f(x) = 0$)?

d. (4 pts) Find all extrema of this function, and classify them.

e. (2 pts) Without a calculation, describe how you know that there is some $c \in (6, \infty)$ such that $f'(c) = -1$.

f. (2 pts) What is the linearization of f at $x = 3$? Use the linearization and one application of Newton's method to estimate a root of f.

g. (10 pts) Discuss the concavity of f over its domain, then graph the function below, incorporating all relevant information:



Problem 2 (10 pts). Choose one of the following two problems:

a. A boy shooting an arrow wants to achieve the greatest distance possible. The angle at which he points the arrow is for him to determine. Assume that

   i. The arrow leaves the bow at a fixed speed, v_0.

   ii. The arrow leaves from ground level ($h = 0$).

   iii. The arrow flies in a parabolic arc (that is, neglect air resistance).

   iv. While the arrow is in the air, it flies with horizontal speed $v_0 \cos(\theta)$.

   v. Its vertical velocity is in opposition to gravity, with initial speed $v_0 \sin(\theta)$: hence, its height h is given by $h(t) = \frac{-g}{2}t^2 + v_0 \sin(\theta)t$.

   Find the angle that maximizes the distance travelled.

b. A rectangular industrial warehouse consisting of three adjacent equal-sized units joined together is to be built. Assume that the wall material costs \$200 per linear foot, and the company allocates \$2,400,000 to the project.

i. What dimensions maximize the total area of the warehouse?

ii. What is the area of each compartment?

Problem 3 (10 pts). This problem concerns $f(t) = 3t + 6\sin(2t)$.

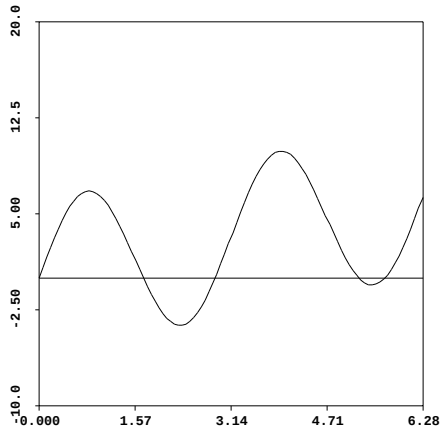
a. (2.5 pts) Determine all anti-derivatives F of f.

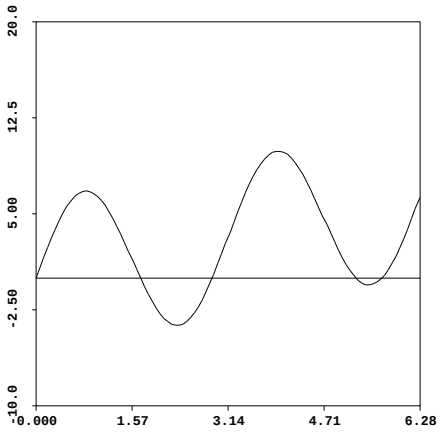
b. (2.5 pts) Find the particular anti-derivative with value $F(0) = -1$.

c. (2.5 pts) What is the graphical difference between members of the anti-derivative family? Explain.

d. (2.5 pts) Where are the extrema of all anti-derivatives on $[0, 2\pi]$?

Problem 4 (10 pts). Use the two copies of the graph of the function $f(t)$ below to answer the following two questions:





a. (6 pts) Estimate the area under the graph on the interval $[0, 2pi]$ in three different ways (indicate on the graph on the left the function values that you're using):

   i. Compute R_4

   ii. Compute L_4

   iii. Compute T_4

b. (4 pts) On the right-hand graph of f trace (as well as you can) its particular antiderivative that takes the value 0 at 0.