

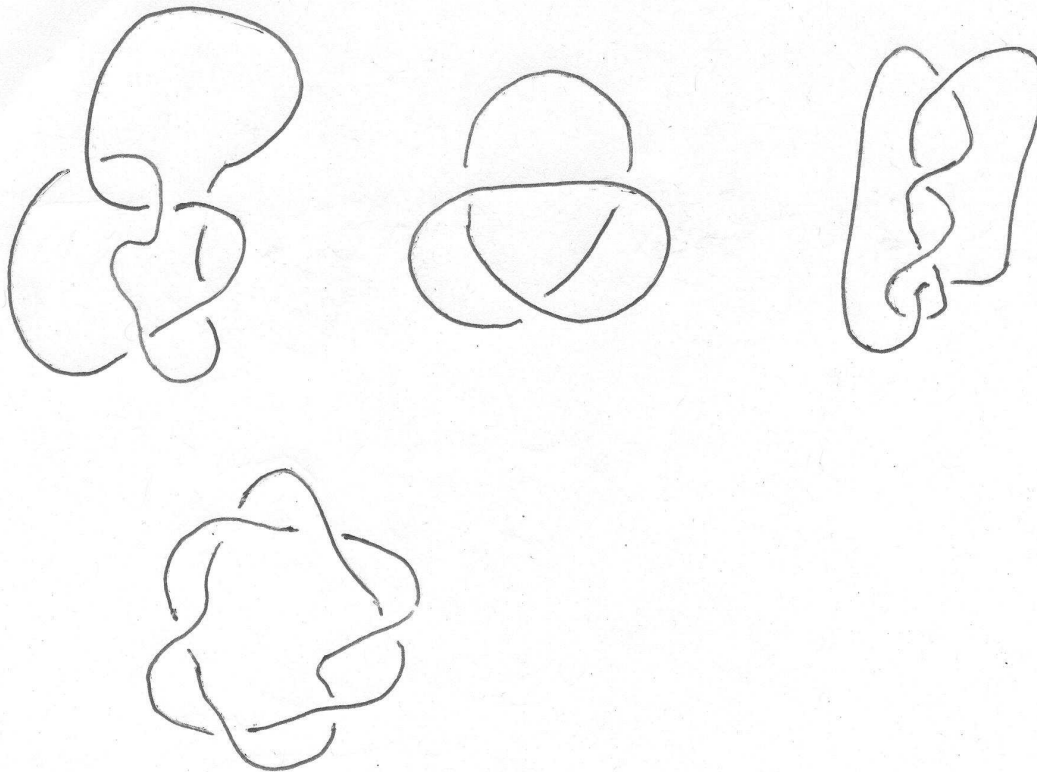
MAT115, Test 3: Platonic Solids, Knots, Bands, Graphs

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1: (10 pts)

a. (8 pts): Identify the following knots/links:



b. (2 pts): Draw the Borromean rings

Problem 2: (10 pts) This problem concerns the Platonic solids.

a. (4 pts) What makes two solids dual? Which solids are duals to which other solids?

b. (3 pts) There is an object on the desk in front associated with a Platonic solid. Come up and take a look at it. What does it have to do with Platonic solids?

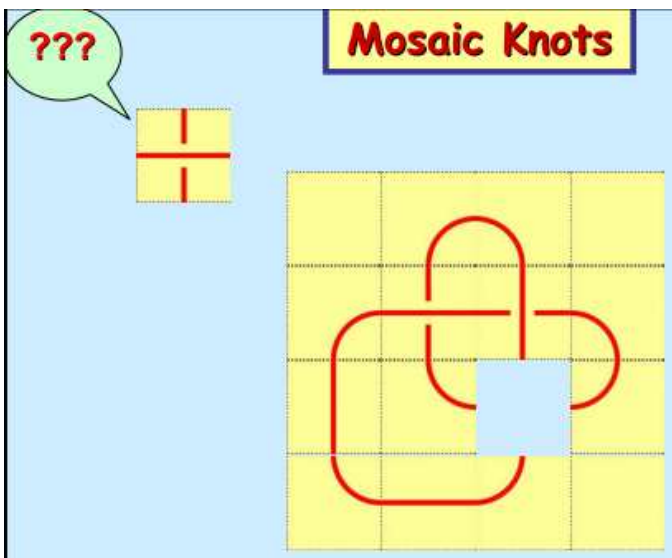
c. (3 pts) Make a drawing of a dual pair of solids, illustrating their dual relationship.

Problem 3: (10 pts)

- a. Circle three spots where each different Reidemeister move (label them I, II, and III) would be appropriate. Label each spot by its move. Show that the knot is not tricolorable, and identify the knot.



- b. In this knot mosaic, two configurations of the crossing tile are possible. What two knots result depending on how the tile is placed?



Problem 4: (10 pts) Draw nice representations (projections) of

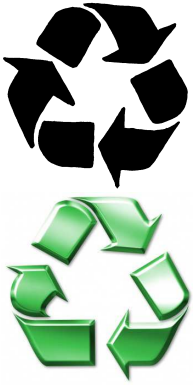
a. The figure-eight knot

b. A trefoil knot

c. One of the five knots (your choice, but identify which you intended to draw).

Problem 5: (10 pts)

- a. (4 pts) Which of these recycling symbols is the true Mobius band (with exactly one half-twist)? Explain your choice!



- b. (2 pts) Describe **precisely** what you get when you cut a Mobius band down the middle.
- c. (2 pts) Describe **precisely** what you get when you cut a Mobius band “into thirds”, hugging an edge $1/3$ of the way in.
- d. (2 pts) What **knot** is associated with a thrice-twisted band (that’s three half-twists)?

Problem 6: (10 pts)

- a. Given: two projections of knots are tricolorable. Circle the number(s) of all that are true:
- i. Neither knot is the unknot.
 - ii. The two must be projections of the same knot.
 - iii. Either could be the trefoil knot.
 - iv. One could be the trefoil knot, and one could be a five knot.

b. Draw “Solomon’s Knot”.

c. Describe the problem that led Euler to develop graph theory.

d. What are the three concepts that make the “Really Cool Object” so cool?

Problem 7: (10 pts) More Graphs:

a. (2 pts) What are the two simplest non-planar graphs, as discussed in class? Draw one of them.

b. (2 pts) Draw a **planar** graph associated with the tetrahedron.

c. (6 pts) Draw all simple graphs of four vertices. Which are dual to each other? (Draw dual pairs next to each other.)