MAT385 Test 1 (Fall 2011): 1.1-1.4; 2.1, 2.2, 2.4

Name:

Directions: Problems are equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1: (10 pts)

a. (5 pts) Using statement letters H, F, and B, express the following as a wff: If you're either happy or friendly, and if you're not broke, then you're either not happy or you're broke.

b. (3 pts) Negate the wff, and write the simplest equivalent wff possible.

c. (2 pts) Construct a truth table for the negated wff above.

Problem 2: (10 pts)

a. (6 pts) Use the following predicates and appropriate quantifiers on the domain of all animals to write the English sentences in predicate wffs:

F(x) – "x is a fish"

C(x) – "x is a carnivore"

E(x,y) – "x eats y"

i. All carnivores eat some fish.

ii. Some carnivores eat all fish.

iii. Not all carnivores eat all fish. (Do **not** simply leave the negation hanging outside....)

b. (4 pts) Use propositional logic to prove that the following argument is valid. Use statement letters S, G, D.

The Sun shines only if the flowers grow. If the flowers grow, then the mouse won't die. The mouse dies. Therefore the Sun didn't shine.

Problem 3: (10 pts) Exactly one of these wff is valid:

a. $(\forall x)[P(x) \to Q(x)] \to [(\exists x)P(x) \to (\forall x)Q(x)]$

b. $(\forall x)[P(x) \to Q(x)] \to [(\exists x)P(x) \to (\exists x)Q(x)]$

Prove the valid wff, and provide an interpretation which disproves the other.

Problem 4: (10 pts)

a. (5 pts) Either prove that this wff is valid or give an interpretation in which it's false:
$(\forall x)(P(x) \vee Q(x)) \wedge (\exists x)Q(x)' \rightarrow (\exists x)P(x)$

b. (5 pts) Using predicate logic, prove this argument valid in the domain of all living things (use $P(x), I(x), D(x, y), H(x)$):

Some people are disgusted by all insects. Some insects are huge. Therefore there is a person who is disgusted by something huge.

Problem 5: (10 pts) Consider the following theorem:

If x is a natural number, and x is divisible by only 2, then $x = 2^n$ for some natural number n.

a. (3 pts) What is the converse of this theorem? Is the converse of this theorem true?

b. (2 pts) What is the contrapositive of this theorem?

c. (5 pts) Demonstrate proof by contradiction to prove the theorem (you may assume that every integer has a unique prime factorization). It isn't hard – just lay it out appropriately.

Problem 6: (10 pts) Use proof by induction to demonstrate that, for $n \geq 2$,

$$1 + r + r^2 + \cdots + r^{n-1} = \frac{1 - r^n}{1 - r}$$

Problem 7: (10 pts)

a. (8 pts) Give a proper recursive definition of all finite **anti-**palindromic strings on an alphabet of two symbols (0 and 1). Anti-palindromic strings are those which read exactly the opposite in the opposite direction: the bits get "flipped" in reading backwards. For example, 0011 is anti-palindromic, as is 101010. Every bit must flip. Every anti-palindromic string looks different when written backwards.

b. (2 pts) How many anti-palindromic strings are there of any given length?