MAT385 Test 2 (Fall 2011): Sets, Graphs, Trees

Name:

Directions: Problems are equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Skip one problem** (and write "Skip" clearly on it). **Good luck!**

Problem 1: (10 pts)

a. Consider the set S of all simple graphs of three vertices or fewer. Draw all seven elements of S, and name them a-g, in order of

   i. increasing number of vertices, and then

   ii. increasing number of edges.

b. Let's create a new graph G, where the vertices are the elements a-g of S. Consider two vertices adjacent if one simple graph can be obtained from another by either

   i. the addition or removal of a single vertex, or

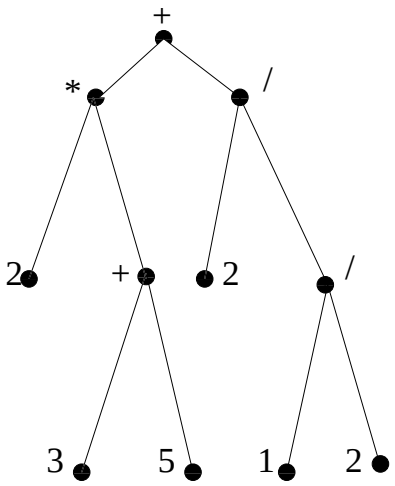
   ii. the addition or removal of a single edge.

   Fill in and describe the corresponding adjacency matrix (let "1" indicate adjacency). Give advantages and disadvantages to storing G in this way.

	a	b	c	d	e	f	g
a							
b							
c							
d							
e							
f							
g							

c. Draw the graph G, and describe its characteristics as best you can.

Problem 2: (10 pts) Consider the following tree:



a. To the right of the tree, give the following traversals:

   i. pre-order

   ii. in-order

   iii. post-order

b. Draw the expression tree for the infix expression $(\sin(x)/3 + 2 * \cos(x))/(2 - x)$. Assume that single children are left-children. Then give the corresponding prefix expression.

c. The postorder list of nodes for another tree is

   l, i, m, h, k, b, a, e, c, o, p, n, d, j, g, f

   The preorder list of nodes is

   f, a, h, l, i, m, b, k, e, g, p, c, o, j, n, d

   Draw the tree.

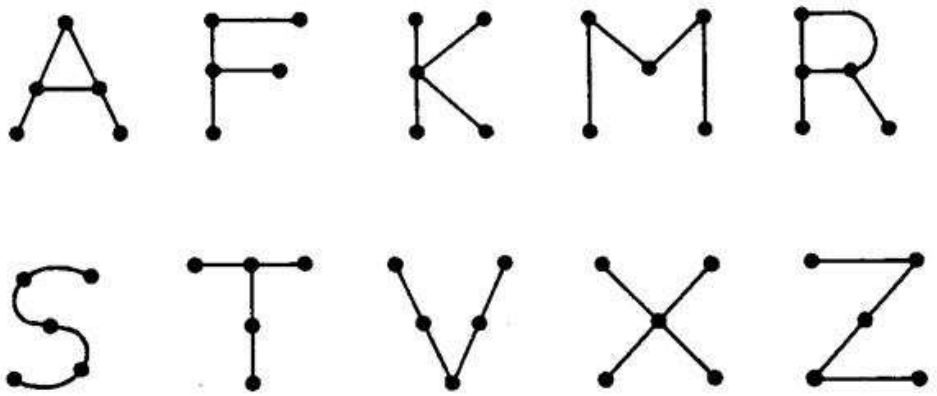
Problem 3: (10 pts)

a. (4pts) Create a binary search tree from this sentence (where capitalization does not count). What is its depth?

b. (4pts) What is the smallest depth possible for a binary tree to contain this data? Re-enter the data so as to create this smaller tree.

c. (2pts) What is the minimal worst-case number of comparisons that a search algorithm would have to do to check whether a word were in our tree or not? Does binary tree search on the tree of part b. satisfy this best worst-case number of comparisons? What would be the worst order in which to enter the data?

Problem 4: (10 pts) Consider the set L containing the following letters:



a. (4pts) Some of the following letters are isomorphic to each other as graphs, and others are not. Create sets containing isomorphic letters.

b. (2pts) Are all distinct non-rooted trees of five vertices represented here?

c. (4pts) Determine whether the following two graphs are isomorphic. Give reasons!

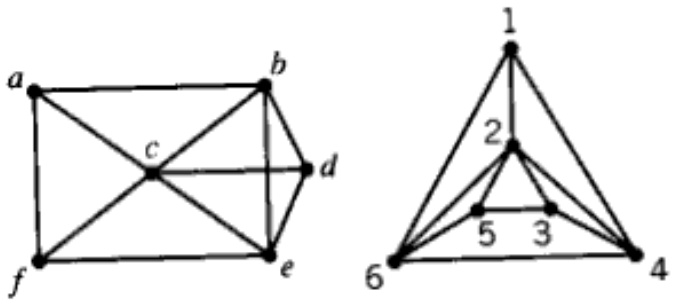
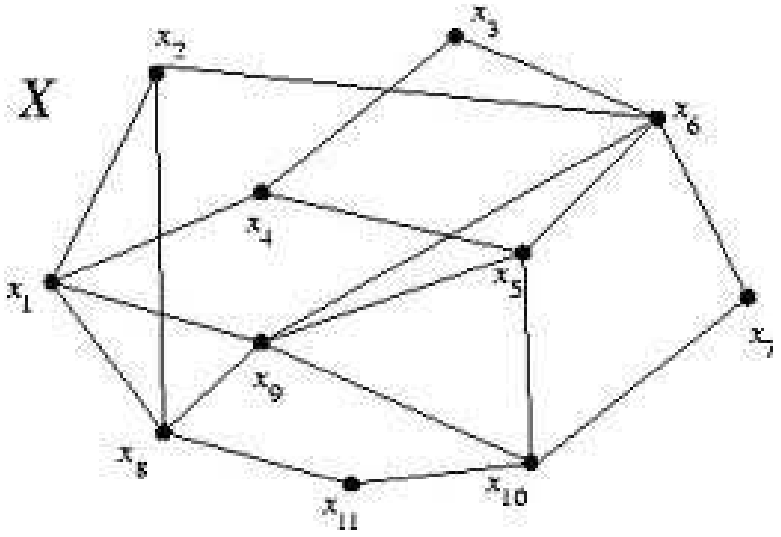


Figure 1: Isomorphic or no?

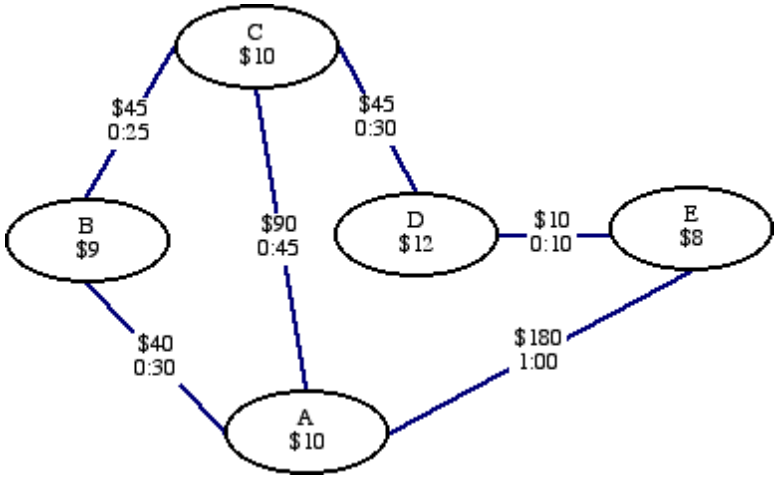
Problem 5: (10 pts)

a. Do a breadth-first and depth-first traversal of the graph on the left in Figure 1, Problem 4. Start from node f, and illustrate your process so that I might follow your logic.

b. Check out the following graph: are either Euler paths or Hamiltonian circuits possible? If so, draw them.



Problem 6: (10 pts) Consider the graph in this figure



a. (5pts) Use Djikstra's algorithm to find the **cheapest** route between E and A, and its total cost.

b. (5pts) Use the Bellman-Ford algorithm to find the **quickest** route between C and all other nodes.