MAT115 Final (Spring 2012)

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). You may skip up to 12 points without penalty – write "skip" CLEARLY on each skipped problem. **Good luck!**

Problem 1: (10 pts) Put the following terms into one-to-one correspondence with the first nine natural numbers, by the order in which they occurred in history ("1" is earliest):

Mathematical Event	**Order of historical introduction**
Fractals	
Pascal's Triangle	
Discovery of zero	
Pythagoreans's discovery of $\sqrt{2}$	
Ishango Bone	
Fibonacci Numbers	
Ramanujan's 1729	
Bridges of Konigsberg problem	
Egyptian Multiplication	

Problem 2: (10 pts) Bases:

a. Write the number 2012_{10} in base 7.

b. Write the number 2012_3 in base 10.

Problem 3: (6 pts) FYI: 1 1 2 3 5 8 13 21 34 55 89 144 233 377 610 987 1597

You and I are playing a game of Fibonacci Nim with coins. In each of the three cases below, we start with the number of coins specified, and you are to

a. explain whether you would prefer to go first or second (and why), and then

b. Give your **first** move assuming that you're in your preferred position. You may have to first say what I would do, assuming that I always use the strategies described in class (including the slow-down defensive strategy).

Warning: Do not play out an entire game! Don't waste your time!

- 37

  a.

  b.

- 233

  a.

  b.

Problem 4: (5 pts) Explain the notion of duality in the Platonic solids (a drawing could be useful) as well as in graphs.

Problem 5: (6 pts) Counting by partition:

a. A farmer has 92 sheep. Show how he can use counting by partition to write down the number of sheep he has. Explain what the answer you get has to do with the binary number representation of the number 92.

b. As priest for the king, your job is to translate the strings the peasants send into numbers of sheep, so that they can be taxed. A peasant reports the string "1, 1, 1, 0, 1, 0, 0, 1." How many sheep does the peasant have? Show your work (for example, draw a tree)!

Problem 6: (10 pts) Graphs:

a. (4pts) Draw the graph of the Bridges of Konigsberg problem, and explain how one knows if an Euler path is possible.

b. (6pts) Consider the simple graphs with four vertices:

   i. Which one is complete?

   ii. Are all of them planar? Explain.

   iii. Relate one of them to a platonic solid (and name that solid!).

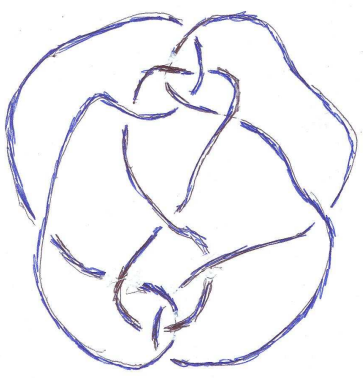
Problem 7: (16 pts) Tidbits:

a. (4 pts) Draw a golden rectangle. What makes it "golden"?

b. (4 pts) What **precisely** do you get if you cut a thrice-twisted band down the middle?

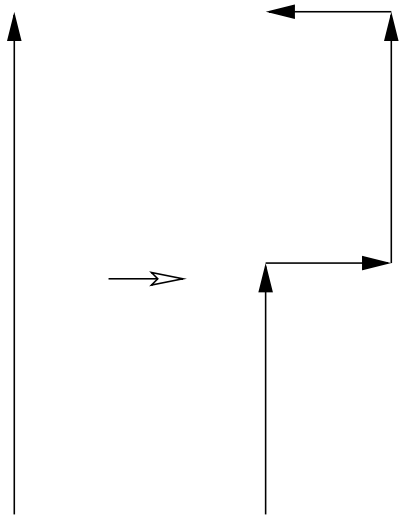
c. (4 pts) How can you use Pascal's triangle to determine the number of different ways that a group of 5 individuals can be distributed between two distinctly different vehicles?

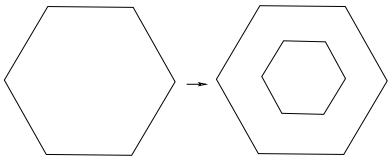
d. (4 pts) What do you make of the object at the front of the class (inspired by Adam's Logo)? Describe it in mathematical terms, as completely as possible.



Problem 8: (10 pts) Fractals.

a. Here I give you two examples of fractal processes (one for arrows, and another for hexagons). You are to draw the next iteration of each (if we "do it again!"). The "rule" is demonstrated visually: the object itself is on the left, and its appearance after the rule is applied is on the right. Draw the next image in the series.





b. Describe (perhaps visually, showing the rule) the

i. Koch snowflake fractal

ii. Sierpinski fractal

Problem 9: (10 pts) Egyptian Multiplication and Division

a. Use Egyptian multiplication to compute the product 56*37.

b. Use Egyptian division to divide 7 loaves among 11 people.

A short 2/n table from the Rhind Mathematical Papyrus

2/3 = 1/2 + 1/6 2/5 = 1/3 + 1/15 2/7 = 1/4 + 1/28

2/9 = 1/6 + 1/18 2/11 = 1/6 + 1/66 2/13 = 1/8 + 1/52 + 1/104

2/15 = 1/10 + 1/30 2/17 = 1/12 + 1/51 + 1/68 2/19 = 1/12 + 1/76 + 1/114

2/21= 1/14 + 1/42 2/23 = 1/12 + 1/276 2/25 = 1/15 + 1/75

2/27 = 1/18 + 1/54 2/29 = 1/24 + 1/58 + 1/174 + 1/232 2/31 = 1/20 + 1/124 + 1/155

2/33 = 1/22 + 1/66 2/35 = 1/30 + 1/42 2/37 = 1/24 + 1/111 + 1/296

2/39 = 1/26 + 1/78 2/41 = 1/24 + 1/246 + 1/328 2/43 = 1/42 + 1/86 + 1/129 + 1/301

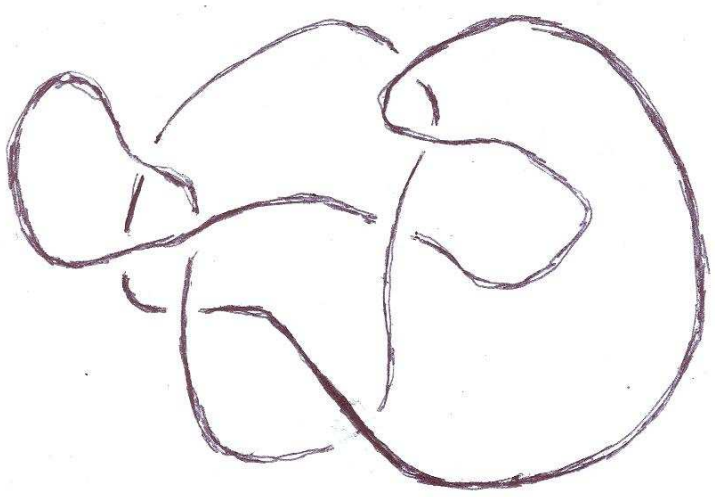
2/45 = 1/30 + 1/90 2/47 = 1/30 + 1/141 + 1/470 2/49 = 1/28 + 1/196

2/51 = 1/34 + 1/102 2/53 = 1/30 + 1/318 + 1/795 2/55 = 1/30 + 1/330

Problem 10: (10 pts) Knots:

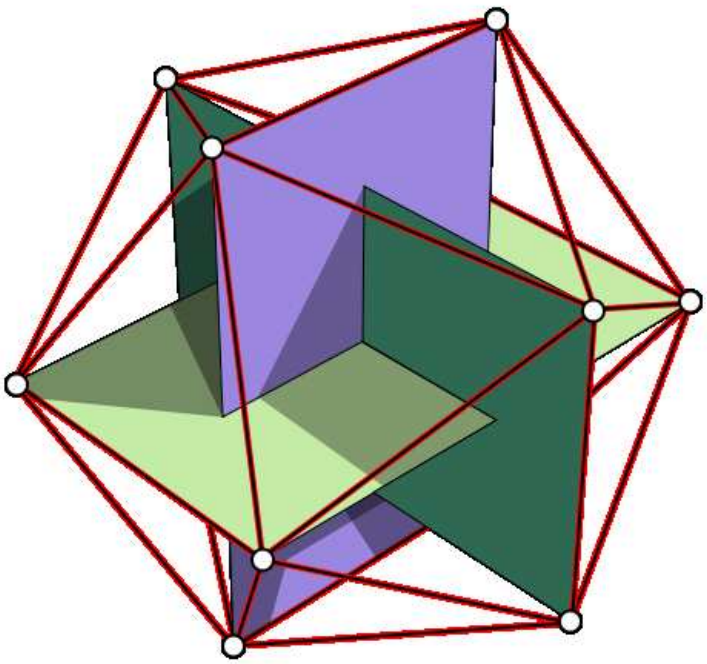
a. Draw a torus knot and draw a twist knot. Name each knot you've drawn.

b. Determine what knot is drawn below. Show Reidemeister moves that are useful by circling places in the knot where you would use each type. Does tricolorability help?



Problem 11: (5 pts) Draw a correct recycling symbol and then the alternative (non-Mobius) symbol. Explain the difference.

Problem 12: (5 pts) Explain the three concepts embedded in the "RCO" (Really Cool Object):



Problem 13: (5 pts) Write the prime factorization of the following. Show your work!

a. 30590

b. 271

Problem 14: (5 pts) Monty Hall offers you a choice of five doors: behind three are donkeys, and behind two others is a new car!

You're invited to choose a door: if you choose a car, then you get to keep it.

Monty opens three doors that you didn't choose: two of them have donkeys, and one has a car. Then he offers you a chance to switch for the fourth door. Do you switch, or stick? Explain!