MAT115 Exam 1 (Spring 2012)

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). FYI: 2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61 67 71 73 79 83 89 97 101 103 107 109 113 127 131. **Good luck!**

Problem 1: (15 pts) Write the following three prime factorizations: if a number is prime, then show what numbers you attempted as factors; if a number is composite, write it as the product of primes in its prime factorization.

a. 17550

b. 1147

c. 829

Problem 2: (10 pts) Counting by partition:

a. You have 73 sheep. Illustrate how to count them by partition: what string of sheep will you report to the king?

b. As priest for the king, your job is to translate the strings the peasants send into numbers of sheep, so that they can be taxed. A peasant reports the string "1, 1, 1, 0, 1, 0, 0, 1." How many sheep does the peasant have? Show your work (e.g. draw a tree)!

Problem 3: (10 pts) Monty Hall strikes again! Now he's got seven doors, behind one of which is a **new car**! The others have donkeys behind them. He gives you a choice of three doors for your own. He then shows you that behind one of the remaining four doors there is a donkey. He asks you if you if you'd like to trade your three doors for the three other unopened doors.

What do you do? Why? What are the exact probabilities of winning, given that you stick and given that you switch?

Problem 4: (10 pts) Short answer:

a. Is the number 1 prime? Explain! [A "yes" or "no" answer alone will earn zero points.]

b. Give two other historical methods of counting discussed in class (besides counting by partition).

c. Give a history of zero. Did the ancient Greeks have it? How about the ancient Egyptians? Who put zero on solid ground, and when?

d. Explain the history and value of the Rosetta stone.

e. For the birthday problem, about how big a crowd in a room gives you the edge when you bet that there will be two people in the room with a common birthday? [You need to get this to within two people!]

Problem 5: (10 pts) This problem concerns the "Great Fraudini" and his card trick:

a. The Great Fraudini is in front of a large audience, delighting yet another crowd with his card trick. He has asked an audience member to choose a number. The audience member writes the number on a blackboard while Fraudini is off stage and then the number is erased. When Fraudini returns the audience member hands him those cards with the chosen number on them. Unfortunately, at just that moment, Fraudini suffers a fatal heart attack and collapses onstage. As his assistant you rush out on stage, push his inert body aside, and attempt to finish the trick. If the cards you pick up are the cards beginning with 4, 8, and 32, then what was the chosen number?

b. (3pts) Which of Fraudini's cards will the number 45 appear on?

c. (4pts) Explain how/why Fraudini's trick works.

Problem 6: (10 pts)

a. If you flip 3 coins, what is the probability that there is both a head and a tail included in the three outcomes?

b. Four people are trapped in an elevator. What is the probability that at least two were born on the same day of the week?

Problem 7: (20 pts) Egyptian Math

a. Use Egyptian Multiplication to multiply 45*122.

b. Divide 4 loaves among 17 people, writing the answer so as to make the ancient Egyptians happy (using only unit fractions). You may use either of our two methods (the unit table, or the doubling/halving table).

Problem 8: (15 pts)

a. Convert 342_{10} to base 5.

b. Convert 742_8 to base 10.

c. Convert 742_8 to base 2.

A short 2/n table from the Rhind Mathematical Papyrus

2/3 = 1/2 + 1/6 2/5 = 1/3 + 1/15 2/7 = 1/4 + 1/28

2/9 = 1/6 + 1/18 2/11 = 1/6 + 1/66 2/13 = 1/8 + 1/52 + 1/104

2/15 = 1/10 + 1/30 2/17 = 1/12 + 1/51 + 1/68 2/19 = 1/12 + 1/76 + 1/114

2/21= 1/14 + 1/42 2/23 = 1/12 + 1/276 2/25 = 1/15 + 1/75

2/27 = 1/18 + 1/54 2/29 = 1/24 + 1/58 + 1/174 + 1/232 2/31 = 1/20 + 1/124 + 1/155

2/33 = 1/22 + 1/66 2/35 = 1/30 + 1/42 2/37 = 1/24 + 1/111 + 1/296

2/39 = 1/26 + 1/78 2/41 = 1/24 + 1/246 + 1/328 2/43 = 1/42 + 1/86 + 1/129 + 1/301

2/45 = 1/30 + 1/90 2/47 = 1/30 + 1/141 + 1/470 2/49 = 1/28 + 1/196

2/51 = 1/34 + 1/102 2/53 = 1/30 + 1/318 + 1/795 2/55 = 1/30 + 1/330