MAT229 Test 1 (Spring 2012): Exponentials, Logs, Inverses, etc.

Name:

Directions: Problems are not equally weighted. Show your work! Answers without justification will likely result in few points. Attempt all problems – some progress is **much** better than no attempt at all. Indicate clearly your final answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1. (20 pts) Variety pack – show work!

a. Re-express the function $f(x) = 3^x$, an exponential of base 3, as an exponential using base e.

b. Solve for x: $\frac{1}{2}\ln(x+1) - \ln(\sqrt{x}) = 1$

c. If $\theta = \tan^{-1}(x)$, what is $\cos(\theta)$ expressed as a function of x, without trig or arc-trig functions? (Hint: draw a triangle.)

d.

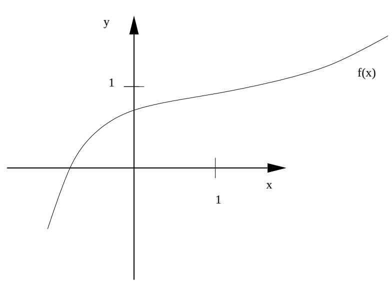


Figure 1: **Carefully** graph the inverse of the function $f(x)$, whose graph is given here.

Problem 2. (15 pts) Given the function

$$f(x) = x\ln(x)$$

a. (5 pts) Use integration by parts to find $\int f(x)dx$.

b. (5 pts) Find an equation of the tangent line to the graph of f at $x = 2$.

c. (5 pts) Can you use L'Hôpital's rule to find $\lim_{x\to 0} f(x)$? If so, do so; if not, explain why.

Problem 3. (15 pts) Suppose that we model a population $P(t)$ of bacteria in plentiful resources as

$$\frac{dP}{dt} = .05P(t) \tag{1}$$

with $P(0) = 2$ (in thousands).

a. (5 pts) Solve this equation for $P(t)$.

b. (10 pts) Suppose that we modify the differential equation (1) to account for crowding:

$$\frac{dP}{dt} = .05(A - P(t)) \tag{2}$$

where A is the so-called equilibrium population. Solve this equation for $P(t)$, with $P(0) = \frac{A}{2}$.

Problem 4. (20 pts) Suppose that a particle is moving with speed (in miles per second)

$$v(t) = \tan^{-1}(t)$$

starting from $t = 0$.

a. (5 pts) What is $\lim_{t \to \infty} v(t)$ (its "terminal" velocity)?

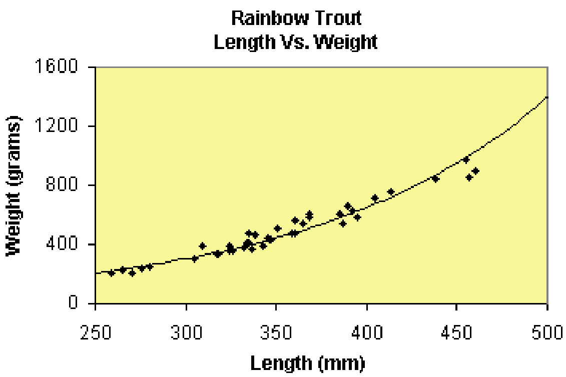
b. (5 pts) Describe the acceleration of the particle. What is its limit as $t \to \infty$? What does this answer tell us?

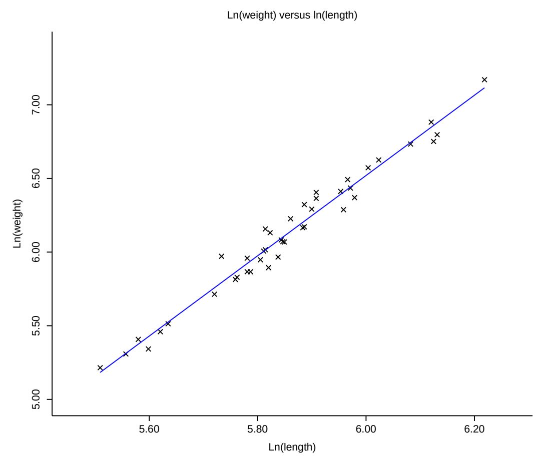
c. (10 pts) How far does the particle move in the first 10 seconds? Give your answer

   a. approximately (correct to the thousandth place), and
   b. exactly.

Problem 5: (15 pts) Rainbow trout were analyzed for heavy metals by the Washington State Department of Ecology. As part of this study, the length (in millimeters) and weight (in grams) of each trout were measured:

Figure 2: At left, length L and weight W data with a graph of an exponential model $W = ae^{kL}$. At right, a plot of $\ln(length)$ versus $\ln(weight)$.





a. (10 pts) Two data points that appear to lie on the graph of the model at left are (247,184) and (455, 975). Estimate the exponential rate constant using these data.

b. (5 pts) Given the plot of $\ln(length)$ versus $\ln(weight)$: The linear model shown has equation $y = 2.72x - 9.83$. If the relationship in the data is well-represented by the model, what is the predicted functional relationship between weight and length? Express it as $W = f(L)$, without using logarithms.