MAT229 Test 2 (Spring 2012): Integration, Sequences, Series

Name:

Directions: Problems are not equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1. (10 pts) Use the definition of an improper integral as a limit to calculate the following integrals (if possible; otherwise conclude that they're divergent). Describe what makes each "improper".

a. $I = \int_0^9 \frac{1}{(x-1)^{2/3}} dx$

b. $I = \int_1^\infty \frac{1}{(x+1)^{5/3}} dx$

Problem 2. (25 pts) Consider the integral

$$I = \int_0^1 e^{-x^2}\, dx$$

There is no closed-form anti-derivative for this integrand. Hence we compute approximations for the integral.

a. (3 pts) Draw a figure that represents I.

b. (6 pts) Compute the left and right endpoint rectangle methods, with 8 sub-intervals. Can we claim that either is a known over- or under-estimate?

c. (3 pts) Show how to compute the trapezoidal method approximation from the previous estimates, and compute it.

Problem 2, cont.

d. (5 pts) Compute the midpoint rectangle methods, with 8 sub-intervals. Draw a graphical representation of this approximation.

e. (3 pts) Show how to compute the Simpson's rule approximation from the previous estimates, and compute it.

f. (5 pts) Compare each estimate to the "true" value (as given by your calculator). Are the results what you would ordinarily expect?

Table 1:

Method	Estimate	Error ("true" minus estimate)
Calculator		NA
LRR		
RRR		
Trapezoidal		
Midpoint		
Simpson's		

Problem 3. (10 pts) A sine function can be used to approximate a football. One needs to simply rotate the function about the x-axis, on the interval $[0, \pi]$.

a. (4 pts) Draw $f(x) = \sin(x)$ on this interval, and write an integral that represents the **arc length** of this portion of the graph of the function f; then compute its value (by whatever means you can).

b. (6 pts) Draw the football on this interval. Now write an integral that represents the **surface area** of the football of this portion of the function f, and use your calculator to compute its value. Compare this value to Simpson's rule with 20 sub-intervals.

Problem 4. (10 pts) Sequences are defined in various ways. For example, one sequence is the Fibonacci numbers, where

$$F_{n+1} = F_n + F_{n-1}$$

with $F_1 = 1$ and $F_2 = 1$.

a. (2 pts) Write the next five terms in the Fibonacci sequence.

b. (2 pts) Consider the ratio of successive terms: $r_n = \frac{F_{n+1}}{F_n}$. Write the first five terms in this sequence.

c. (6 pts) Assuming that this sequence $\{r_n\}$ converges, find its limit.

Problem 5. (20 pts) Determine whether the following series converge or not. Explain the test you're using, and show that the series meets the criteria for the use of that test. Then show the results of applying the test.

a. $\sum_{n=1}^{\infty} \frac{1}{F_n}$ (where $\{F_n\}$ is the Fibonacci sequence).

b. $\sum_{n=1}^{\infty} \frac{(-1)^n}{F_n}$

Problem 5, cont.

c. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$

d. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

Problem 6. (10 pts) A pollutant is found in a well. In this event, one strategy is to "pump and treat" the well – that is, pump out the well mixture and draw in clean water to replace the polluted material. The polluted material is treated and disposed of elsewhere.

Assume that the well has a capacity of 10,000 gallons, and an initial pollutant concentration of $c_0 = 4000$ ppm (parts per million – i.e. the ratio of pollutant to total is 4000/1000000, or 1/250. Assume that fresh water is coming in at a rate of one gallon per minute.

Assume that the incoming water is immediately well mixed with the mixture in the well (which is initially 9,999 gallons of polluted water). Assume that the gallon of clean water is added to the existing mixture, and that a gallon of the resultant mixture is removed at the end of the minute.

a. What's the concentration c_n of the pollutant (the fraction of pollutant to total material) at minute n?

b. How many minutes until the concentration of the pollutant falls to less than 1 ppm? How many days is that?