

MAT229 Test 3 (Spring 2012): Series and Vectors

Name:

Directions: Problems are not equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1. (15 pts) Power series.

a. (3 pts) For any power series, there are only three distinctly different possibilities for the radius of convergence. What are they?

b. (12 pts) For the following two functions, find a power series about $a = 0$, and find the radius of convergence:

a. $f(x) = \frac{3}{16 + x^2}$

b. $f(x) = \frac{1}{(1 + x)^2}$

Problem 2. (10 pts) Find the radius of convergence and interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{n^2 x^n}{2 \cdot 4 \cdot 6 \cdots (2n)}$$

Problem 3. (10 pts) Find a power series representation for $I = \int_a^b \frac{dt}{1+t^3}$. For what limits of integration a and b would this representation be valid (that is, equal to I), given your power series?

Problem 4. (10 pts) Use the Maclaurin Series for e^x to compute $\frac{1}{e^2}$ to within .001 of its true value. Exactly how many terms do you need to use?

Problem 5. (10 pts) Write the Taylor series of $f(x) = \sin(x)$ expanded about $a = \frac{\pi}{4}$. In particular, give the exact value of the n^{th} term, a_n .

Problem 6. (10 pts total) a. (6 pts) Draw a realistic coordinate system (remember labels?), and carefully and correctly plot the points $(1,-2,3)$, $(1,1,1)$, and $(3,0,2)$.

b. (4 pts) In your picture above, draw the triangle containing the three points and then compute the distance between $(1,-2,3)$ and $(3,0,2)$.

Problem 7. (5 pts) Given $\bar{v} = \langle 1, 2, 3 \rangle$.

a. Find a vector that has the same direction as \bar{v} but length 2.

b. A vector orthogonal to \bar{v} .

Problem 8. (5 pts) A woman walks due west on the deck of a ship at 5 mi/h. The ship is moving north at a speed of 12 mi/h. Find the speed and direction of the woman relative to the surface of the water.

Problem 9. (10 pts) A tow truck drags a stalled car along a road. The chain makes an angle of 30° with the road and the tension in the chain is 1500 N. How much work is done by the truck in pulling the car 1 km?