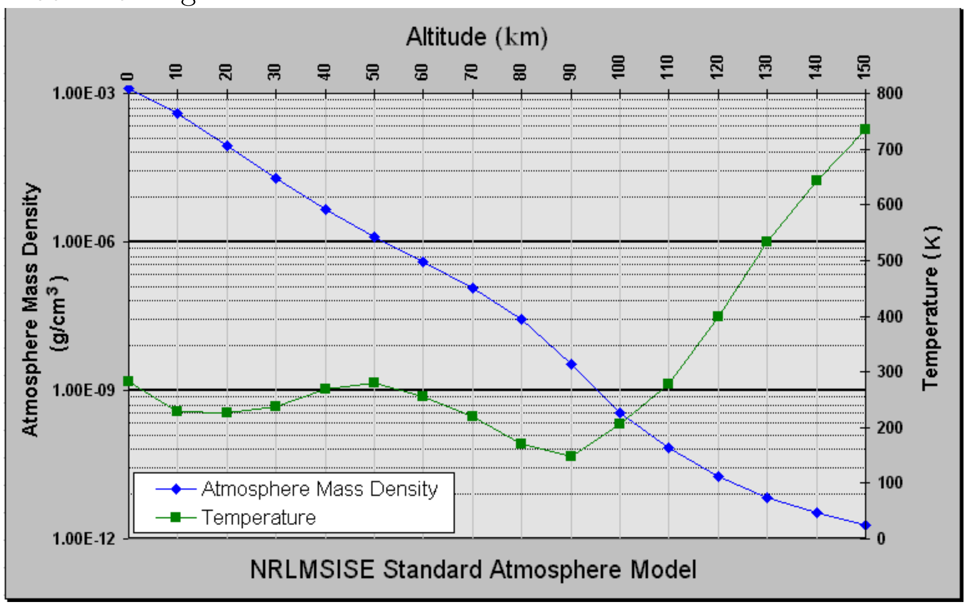
Exponential Functions Lab

Figure 1: `http://upload.wikimedia.org/wikipedia/commons/d/de/Atmosphere_model.png`; According to the National Center for Atmospheric Research, "The total mean mass of the atmosphere is 5.1480 $\times 10^{18}$ kg...."



Notice the strange scale for the mass density. With the y-axis changing in that funny way (by powers of 10), the graph of the mass density looks linear. [Remember that mathematicians love linear models – they're the simplest really useful models.]

a. The thing changing linearly along the y-axis is the **exponent** of a power of ten, in scientific notation. Atmospheric mass density seems to be falling **linearly** in this strange scale.

   Write a function for y – the "exponent of the power of ten" of the atmospheric mass-density – as a function of altitude h. Since it's easy to write equations for straight lines given two points, let's pick (0,-3) and (50,-6).

b. Describe then how the atmospheric mass density function in the graph of Figure 1 can be modelled by

$$\rho(h) = 10^{-3(1+h/50)}$$

This is an example of an exponential function (although it may look kind of strange as given above). Let's re-express it, using properties of exponentials: show that we can write ρ as $\rho(h) = \alpha 10^{-3h/50}$. [What is α?]

c. Finally, then, we can construct an integral to compute the mass of the atmosphere: it involves some unusual elements that we will discuss:

$$Mass = \int_0^\infty \rho(h) 4\pi (h + 6371)^2 dh$$

where 6371km is the radius of the Earth.

i. What conversions do you have to do in your calculations to get the mass in kilograms?

ii. What do you make of having infinity in your integral? We'll talk more about these "improper integrals" down the road.