Section Summary: 10.4

a. Definitions

None.

b. Theorems

None.

c. Properties/Tricks/Hints/Etc.

The area of a region created by positive continuous function f in the equation $r = f(\theta)$ as θ ranges over $[a, b]$ is

$$A = \int_a^b \frac{1}{2}[f(\theta)]^2 d\theta$$

$(0 < b - a \leq 2\pi)$.

To find *all* points of intersection of two polar curves, it is recommended that you draw the graphs of both curves.

The arc length formula is derived by treating a polar curve as a parametric equation, and then simplifying: so if $r = f(\theta)$, then

$$x = f(\theta)\cos\theta$$
$$y = f(\theta)\sin\theta$$

and

$$L = \int_a^b \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} d\theta$$

which, when simplified using the equations for x and y above, yields

$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$$

d. Summary

This section is more of the same-ol' same-ol': finding areas and arc lengths of parametric curves, only using somewhat unusual tactices. For example, we used sectors of circles to compute the area, whereas for arc length we simply rewrote polar equations as parametric equations, and applied the formula for arc length in that case (later simplified to eliminate x and y).

The area problem is a little different than the standard area problem, in that we're looking for the area of a sector as an angle changes - as a piece of pie, rather than as a traditional piece of iced cake, as it were. We aren't sitting squarely on the x-axis, but are focused at the pole.