**Section Summary: The Integral Test**

# 1 Theorems

Suppose $f$ is a continuous, positive, decreasing function on $[1, \infty)$ and let $a_n = f(n)$. Then the series

$$s = \sum_{n=1}^{\infty} a_n$$

is convergent if and only if the improper integral

$$I = \int_1^{\infty} f(x)dx$$

is convergent. If $I$ diverges, then $s$ diverges (and *vice versa*).

Note that it is not essential for $f$ to be positive and decreasing everywhere, but it must be *ultimately positive and decreasing* (that is, decreasing beyond some fixed value of $x$).

The **p-series**

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

is convergent if $p > 1$, and divergent if $p \leq 1$.

If $\sum a_n$ converges by the integral test, and the **remainder** $R_n = s - s_n$, then

$$\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_n^{\infty} f(x)dx$$

(this gives us a bound on the error we're making in the calculation of a series).

# 2 Summary

Integrals serve as useful tools for evaluating series, determining whether series exist, and estimating the error we're making in an estimate of the limit of a series.