Section Summary: Alternating Series

1 Definitions

Alternating Series: a series whose terms are alternately positive and negative.

2 Theorems

Alternating series test: If the alternating series

$$\sum_{n=1}^{\infty}(-1)^{n-1}b_n = b_1 - b_2 + b_3 - \dots$$

with $b_n > 0$ satisfies

    a. $0 \le b_{n+1} \le b_n$ for all n, and

    b. $\lim_{n \to \infty} b_n = 0$

then the series is convergent.

    Alternating Series Estimation Theorem: If $s = \sum(-1)^{n-1}b_n$ is the sum of an alternating series that satisfies the conditions of the alternating series test, then
$$|R_n| = |s - s_n| \le b_{n+1}$$

3 Summary

Alternating series have nice cancellation properties, encaptured in the alternating series test and the alternating series estimation theorem. The error term is especially nice: if we stop at the nth partial sum, then the magnitude of the error is less than the first neglected term.

    The author takes pains to warn us not to apply this rule to a general series! Only those series whose terms alternate need apply....