Section Summary: Power Series

1 Definitions

power series: A **power series** is a series of the form

$$\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + \dots$$

where the c_i are called the **coefficients** of the series. More generally,

$$\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \dots$$

is called a **power series in** $x - a$, or a **power series centered at** a, or a **power series about** a.

Consider a power series about a: the **radius of convergence** is the largest number R such that the power series converges for all x in

$$|x - a| < R$$

The endpoints often need to be tested separately, and the resultant **interval of convergence** is the interval for which the power series is defined (possibly including one or both of the endpoints).

2 Theorems

For a given power series

$$\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \dots$$

there are only three possibilities:

a. The series converges only when $x = a$ (radius of convergence $R = 0$);

b. the series converges only when $|x - a| < R$ (radius of convergence R), and possibly at the endpoints, and diverges otherwise; or

c. the series converges for all real numbers x (radius of convergence $R = \infty$).

3 Properties, Hints, etc.

"In general, the Ratio Test (or sometimes the Root Test) should be used to determine the radius of convergence R. The Ratio and Root Tests always fail when x is an endpoint of the interval of convergence, so the endpoints must be checked with some other test." (p. 768)

4 Summary

We can think of power series as infinite polynomials: very oddly defined functions! Of course we'll wonder about the domain of these functions, which is fundamentally tied to the issue of convergence. We'll say that these functions are defined for any value of x for which the series converges.

We'll use these functions to provide useful approximations to functions (by only taking a partial sum), meaning that it will be possible to approximate functions by **finite** – and hence ordinary – polynomials.

If we want to approximate a function in the vicinity of a certain point a, then we will center our power series about a.