**Section Summary: 11.9 – Functions as Power Series**

# 1 Definitions

# 2 Theorems

**term by term differentiation and integration** If the power series

$$\sum c_n(x-a)^n$$

has radius of convergence $R > 0$, then the function $f$ defined by

$$f(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + \ldots = \sum_{n=0}^{\infty} c_n(x-a)^n$$

is differentiable (and therefore continuous) on the interval $(a-R, a+R)$ and

a.

$$f'(x) = c_1 + 2c_2(x-a) + \ldots = \sum_{n=0}^{\infty} nc_n(x-a)^{n-1}$$

b.

$$\int f(x)dx = C + c_0(x-a) + c_1\frac{(x-a)^2}{2} + \ldots = C + \sum_{n=0}^{\infty} c_n\frac{(x-a)^{n+1}}{n+1}$$

The radii of convergence of the integrated and differentiated power series are both $R$.

# 3 Properties, Hints, etc.

Notice the "Leibniz formula" for $\pi$ (p. 774): since it's an alternating series, we find $\pi$ to any accuracy we desire, by simply making the first neglected term small enough.

# 4 Summary

We can construct new power series from old ones in several ways: by

a. writing them as composite functions,

b. integration

c. differentiation

The radius of convergence in the last two cases stays the same as the original power series; in the case of composition, we need to recalculate the radius of convergence.

The interval of convergence does not necessarily remain the same, however, so you still have to check the ends separately.

Notice how power series can be used to integrate (approximately) complicated function, and then provide a mechanism for determining the error in the approximation.