Section Summary: Three-Dimensional Coordinate Systems

1. Definitions

- **right-hand rule:** Curl the fingers of your right hand around the z-axis with your thumb pointing in the positive z-direction, then open your hand 90°: your fingers point in the positive x-direction; closed, they point in the positive y-direction.
- **coordinate planes:** analogous to the coordinate axes in 2-dimensions: the surfaces created when one of the three coordinates is set to zero. Each coordinate plane will contain two of the three coordinate axes.
- **octants:** space is now divided into 8 sections, by the walls of the coordinate planes.
- **three-dimensional coordinate system:** Once coordinate axes are established, coordinates (x, y, z) indicate a point in space. There is a one-to-one correspondence between space points and these triples.
- **coordinates of a point:** an ordered triple of the values (x, y, z) indicating the location of the point in the three-dimensional coordinate system.

2. Theorems

- **Distance formula in three-dimensions**: is a simple analogue of the formula in two dimensions. The distance $|P_1P_2|$ between the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is given by
$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
- **Equation of a Sphere:** From the distance formula, we can deduce the equation for the sphere centered on $C(h, k, l)$ of radius r. The sphere is all points equidistance from the center, of distance r, which is indicated by
$$r^2 = (x - h)^2 + (y - k)^2 + (z - l)^2$$

3. Properties/Tricks/Hints/Etc.

Drawing axes is important: see the plots on the following page for right and wrong.

4. Summary

This section serves as an introduction to 3-dimensional coordinate systems and the representation of space, rather than the plane. We see that some formulas extend in quite a natural way.

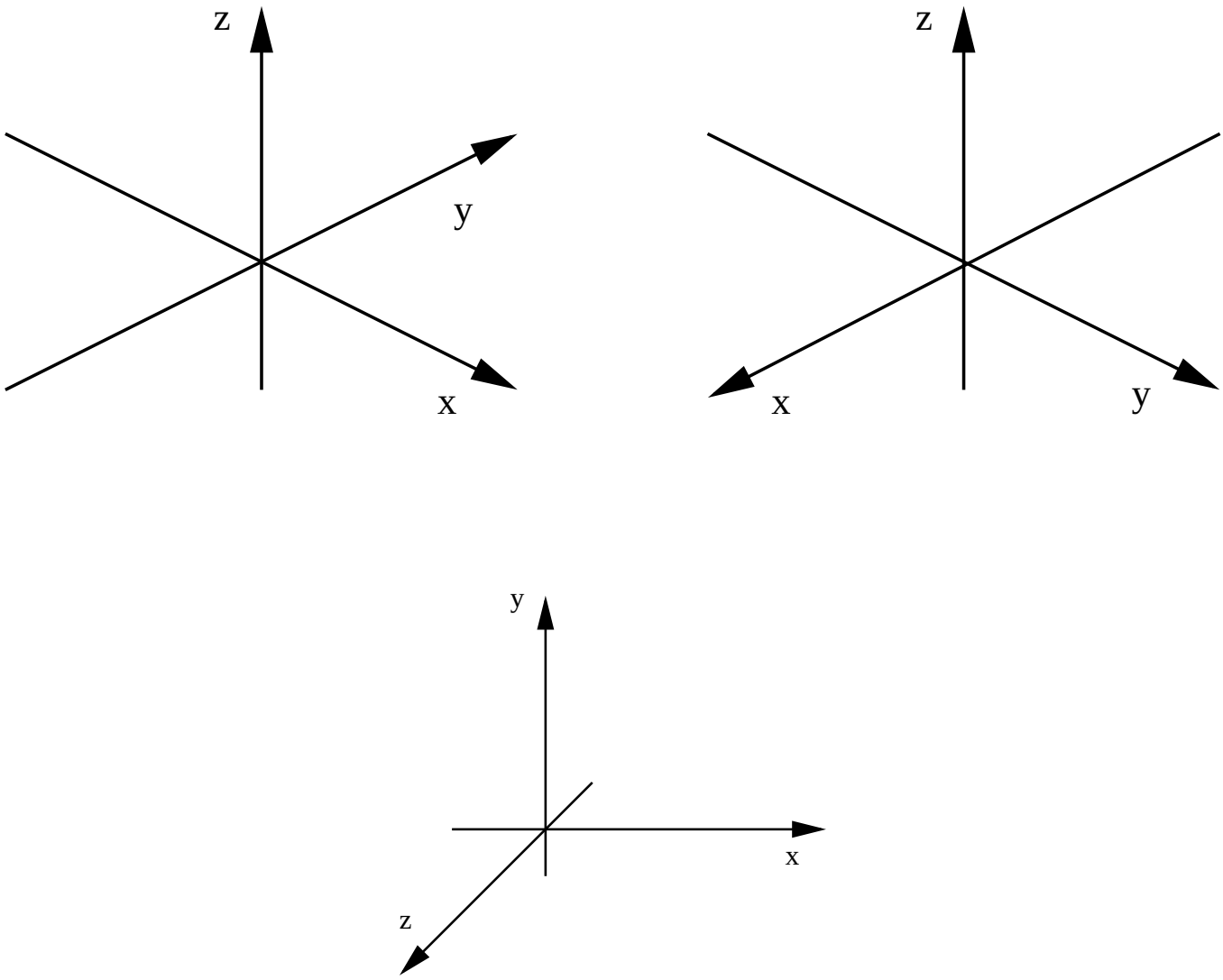

Figure 1: Top: two legitimate representations; Bottom: sin and sacrilege! Don't do it! It's not physically realizable.