Section Summary: 12.2

1. **Definitions**

   - **vector**: a quantity having both direction and magnitude (length). A **two-dimensional vector** is an ordered pair **a**= $\langle a_1, a_2\rangle$; A **three-dimensional vector** is an ordered triple **a**= $\langle a_1, a_2, a_3\rangle$. The numbers a_1, a_2, and a_3 are called **components of a**.

   - A **representation** of the vector **a**= $\langle a_1, a_2\rangle$ is a directed line segment **AB** from **any** point $A(x, y)$ to the point $B(x + a_1, y + a_2)$. A particular representation of **a** is the directed line segment **OP** from the origin to the point $P(a_1, a_2)$, called the **position vector** of the point $P(a_1, a_2)$. Representations in 3-d are defined analogously.

     Vectors will generally be written with an arrow, e.g. **a**, since it's hard to write boldface on paper or on the board.

   - **parallel vectors**: if two vectors are parallel, then one can be written as a scalar multiple of the other:
     $$\mathbf{a} = c\mathbf{b}$$

   - **unit vector**: a vector whose length is 1.

2. **Theorems**

   None to speak of.

3. Properties/Tricks/Hints/Etc.

   - Given the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$, the vector **a** with representation **AB** is
     $$\mathbf{a} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1\rangle$$

   - The length of the two-dimensional vector **a**= $\langle a_1, a_2\rangle$ is
     $$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2}$$
     The length of the three-dimensional vector **a**= $\langle a_1, a_2, a_3\rangle$ is
     $$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

- Addition is defined componentwise, so that $\mathbf{a} + \mathbf{b}$ is defined as

$$\mathbf{a} + \mathbf{b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$$

  Subtraction is defined in the obvious way.

  Addition is carried out geometrically by putting the tail of vector $\mathbf{b}$ to the head of $\mathbf{a}$ and creating the vector from the tail of $\mathbf{a}$ to the head of $\mathbf{b}$, creating a parallelogram.

- Multiplication of a vector by a scalar: If c is a scalar (e.g., a real number) and $\mathbf{a}= \langle a_1, a_2 \rangle$, then the vector $c\mathbf{a}$ is defined by

$$c\mathbf{a} = \langle ca_1, ca_2 \rangle$$

  and similarly for three-dimensional vectors.

- Properties of vectors:
  1. $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$
  2. $\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$
  3. $\mathbf{a} + \mathbf{0} = \mathbf{a}$
  4. $\mathbf{a} + -\mathbf{a} = \mathbf{0}$
  5. $c(\mathbf{a} + \mathbf{b}) = c\mathbf{b} + c\mathbf{a}$
  6. $(c + d)\mathbf{a} = c\mathbf{a} + d\mathbf{a}$
  7. $cd\mathbf{a} = c(d\mathbf{a})$
  8. $1\mathbf{a} = \mathbf{a}$

- Special vectors (the **standard basis vectors**, of length 1):

$$\mathbf{i} = \langle 1, 0, 0 \rangle \qquad \mathbf{j} = \langle 0, 1, 0 \rangle \qquad \mathbf{k} = \langle 0, 0, 1 \rangle$$

  Any vector can be expressed as a sum of the standard unit vectors:

$$\mathbf{a} = \langle a_1, a_2, a_3 \rangle = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$$

- Vectors can be defined in n-dimensions in entirely analogous ways.

4. Summary

This section simply introduces us to a quantity, called a vector, which allows us to capture both magnitude and direction. This is useful (for example to indicate wind speed and direction on a weather map), and a set of rules and properties are defined to help us to manipulate these quantities.

Every vector can be expressed as a sum of special “basis” vectors, which are of unit size (length 1).