Section Summary: Equations of Lines and Planes

1. Definitions

The vector equation of a line L is

$$\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$$

where t is a scalar. Suppose that $\mathbf{r} = \langle x, y, z \rangle$, $\mathbf{r}_0 = \langle x_0, y_0, z_0 \rangle$, $\mathbf{v} = \langle a, b, c \rangle$; since this is true component-wise, we have that

$$x = x_0 + at \qquad y = y_0 + bt \qquad z = z_0 + ct$$

These are the parametric equations of the line L.

These equations are not unique to a line: any point $\mathbf{r}_0$ and vector v with orientation along the line will give another set of equations.

We can solve for t in each of the three parametric equations above to get a set of three equations

$$\frac{x - x_0}{a} \qquad \frac{y - y_0}{b} \qquad \frac{z - z_0}{c}$$

called the symmetric equations of L.

Skew lines are lines that do not intersect and are not parallel. Note: this can't happen in the plane! It only happens in three-space (or higher), that lines can pass like ships in the night....

The parametric equation of a plane P is

$$\mathbf{r} = \mathbf{r}_0) + s\mathbf{v}_1 + t\mathbf{v}_2$$

The vector equation of a plane P is

$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$$

or

$$\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$$

Suppose that $\mathbf{r} = \langle x, y, z \rangle$**,** $\mathbf{r}_0 = \langle x_0, y_0, z_0 \rangle$**,** $\mathbf{n} = \langle a, b, c \rangle$**; then**

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

This is the scalar equation of the plane P with normal vector n.

These equations are not unique to a plane: any vector $\mathbf{r}_0$ **and vector n normal to the plane will give another set of equations.**

By collecting terms in the scalar equation above we find that

$$ax + by + cz + d = 0$$

where $d = -(ax_0 + by_0 + cz_0)$ **This is called a linear equation in** x**,** y**, and** z**.**

Two planes are parallel if their normal vectors are parallel.

2. Theorems

None to speak of.

3. Properties/Tricks/Hints/Etc.

Lines in 3-space (or higher) can pair up in only one of three ways:

- **parallel**
- **intersecting**
- **skew (missing each other completely yet not oriented in the same way)**

Distinct planes in 3-space (or higher) can pair up in only two ways:

- **parallel**
- **intersecting in a line**

The line of intersection can be found by solving both scalar equations of the planes simultaneously.

4. Summary

In the first part of we section, we are introduced to various ways of thinking about lines in space. We meet with several different equations of space lines, and see that lines in space behave a little differently than lines in the plane.

In the second part of the section, we are introduced to various ways of thinking about planes in space. We meet with several different equations of planes.