Section Summary: Separable Equations

In this section we take a look at one type of differential equation that is somewhat analytically accessible. This is another application of differentiation and integration.

We've already met differential equations in the form of the special property of exponential functions: these functions satisfy the equation $f'(x) = cf(x)$.

In the case of the exponential functions, we could make this first-order equation (first-order, because it only involves first-derivatives) pop out from the limit definition of the derivative.

Now we consider more general equations.

(I) Definitions

A **separable equation** is a first-order differential equation in which the expression for dy/dx can be factored as a function of x times a function of y:

$$\frac{dy}{dx} = g(x)f(y)$$

In the case of the exponential equation mentioned above, we would have $g(x) = 1$, and, with $y = f(x)$, we write $f(y) = cy$. The equation then becomes

$$\frac{dy}{dx} = cy$$

and the solution is exponentials.

For the general separable equation, the solution is to re-organize the terms as

$$\frac{dy}{f(y)} = g(x)dx$$

(provided $f(y) \neq 0$). Then we integrate both sides:

$$\int \frac{dy}{f(y)} = \int g(x)dx$$

On the left side we'll have "y-stuff", and on the right we'll have "x-stuff", so y will be given as an implicit function of x. Actually, we'll get a family of functions, because of the arbitrary constant of integration. Example 1, p. 619 illustrates this well.

(II) Properties/Tricks/Hints/Etc.

There are some interesting examples included in this section, such as the "orthogonal families" and the mixing problem. These are non-trivial applications. Don't get lost in the details, but try to appreciate the applications of integration and differentiation.

(III) **Summary**

Differential equations is a separate course, which one takes once the calculus is under one's belt. It's tremendously interesting and useful, and we see some examples of why we'd want to study them in this section.

The first important differential equation we encountered came out of the exponential functions, of the form $f'(x) = cf(x)$. This (first-order) equation has exponential solutions.

Now we expand our arsenal by considering more general differential equations, but solve them by

i. separating the equations into two separate integrals,

ii. solving them separately, and then

iii. defining y implicitly as a function of x.