MAT229 Test 1 (Fall 2014): Inverses, Exponentials, Logs, etc.

Name:

Directions: Problems are not equally weighted. Show your work! Answers without justification will likely result in few points. Attempt all problems – some progress is **much** better than no attempt at all. Indicate clearly your final answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1. (30 points total) Show work!

a. Integrate $\int xe^{-x}dx$

b. Find $\lim\limits_{x\to\infty} x^2 e^{-x}$.

c. Integrate $\int \sin(x)\cos(x)dx$ by parts.

Problem 2. (20 points total)

a. (7 pts) Use the definition of $\sinh(x)$ to find its inverse function $\sinh^{-1}(x)$.

b. (7 pts) Use the general formula for the derivative of an inverse function to find $[\sinh^{-1}(x)]'$, the derivative of $\sinh^{-1}(x)$. You might need the identity $\cosh^2(x) - \sinh^2(x) = 1$.
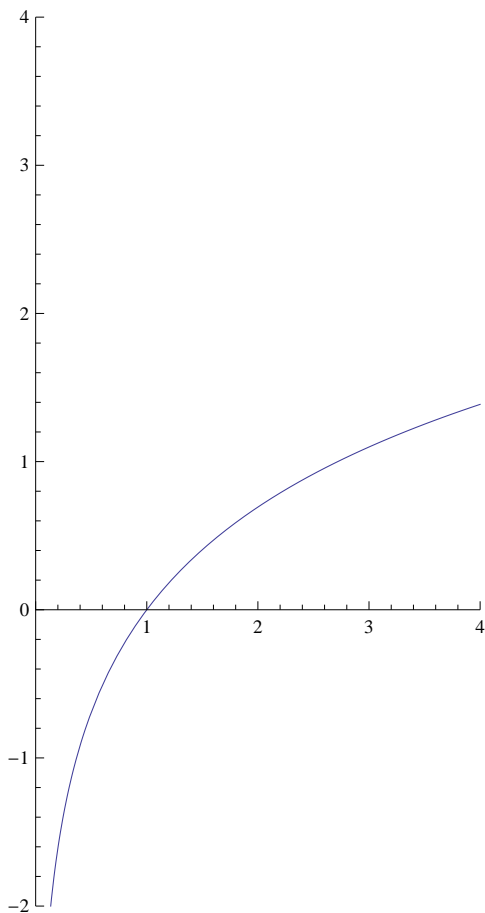
c. (3 pts) Before we can define an inverse for $\cosh(x)$, what must we do?

d. (3 pts) What do we call the graph of $\cosh(x)$? Where do we see it in the world?

Problem 3. (20 points total) Let $f(x) = 2\ln(x - 2)$.

a. (6 pts) Starting from the graph of $\ln(x)$ (below), explain how to obtain the graph of f. Carefully draw $f(x)$ in the coordinate system below, alongside $\ln(x)$.

b. (8 pts) Find the value of x at which the graphs of $f(x)$ and $\ln(x)$ intersect.

c. (6 pts) Find the equation of the tangent line to the graph of f at the point $x = 3$, and add the line to the graph.

Problem 4. (20 points total)

a. (10 pts) Consider $f(x) = \sin(x)$, properly restricted so that it is one-to-one, and hence invertible. What is its domain? What is its range?

Carefully draw $f(x)$ and $f^{-1}(x)$ in the coordinate system below. Label your graph well.
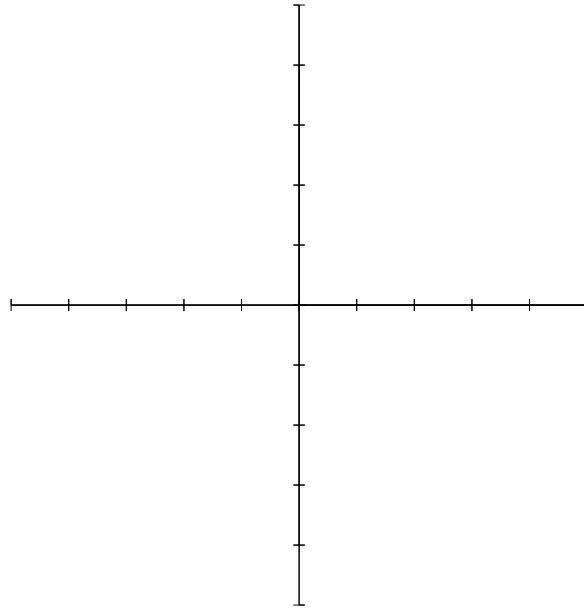
b. (10 pts) Compute the derivative of x^x by any method.

Problem 5: (10 points total)

a. (7 pts) Let $f(x) = b^x$. Use the limit definition of the derivative to show that $f'(x) = cf(x)$, where c is a constant.

b. (3 pts) How do we interpret c? What is its significance?

Extra Credit (5 points): If $\theta = \tan^{-1}(x)$, what is $\cos(\theta)$ expressed as a function of x, without trig or arc-trig functions?