MAT229 Test 2 (Fall 2014): Integration, Parametrics, Sequences

Name:

Directions: Problems are not equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

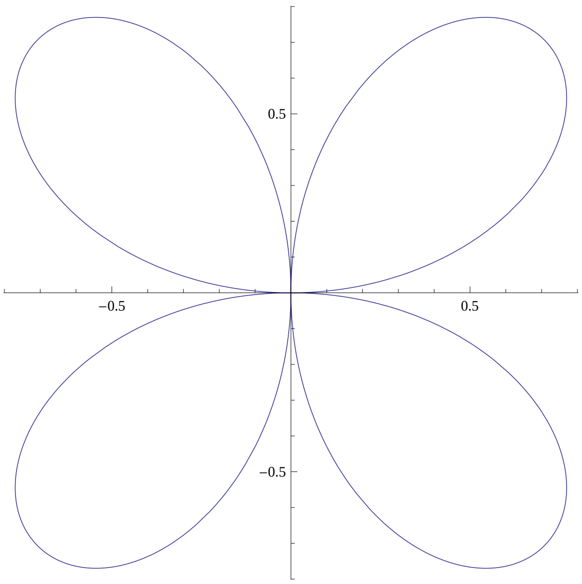
Problem 1. (15 pts) Are the first two integrals convergent or divergent? For those which are convergent, use the definition of an improper integral as a limit to calculate their values. Describe what makes each "improper".

a. $I = \int_1^\infty \frac{1}{(x-1)^{3/2}} dx$

b. $I = \int_0^2 \frac{1}{(x-1)^{2/3}} dx$

c. Show that $\int_1^\infty \frac{1+e^{-x^2}}{x} dx$ is divergent by comparison.

Problem 2. (20 pts) Consider the graph of the curve of the polar function $r(\theta) = \sin(2\theta)$:



a. (10 pts) Represent the area of one petal of this graph as an integral. Justify your limits of integration.

b. (10 pts) Compute the exact value of this integral.

Problem 3. (25 pts) Consider the integral

$$I = \int_1^5 \frac{1}{x} dx$$

a. (5 pts) Compute the exact value of this integral.

b. (15 pts) Compute the Simpson's rule approximation S_4, by first computing the right, left, trapezoidal, and midpoint approximations. Show your work!

Method	Estimate	Error (true minus estimate)
True		NA
LRR		
RRR		
Trapezoidal		
Midpoint		
S_4		

c. (5 pts) Examine the errors. Are the results consistent with what we should expect? (**Be specific.**)

Problem 4. (15 pts) Sequences are defined in various ways. For example, one sequence is the Fibonacci numbers, where

$$F_n = F_{n-1} + F_{n-2}$$

with $F_1 = 1$ and $F_2 = 1$, and $n \geq 3$.

a. (3 pts) Write the next five terms in the Fibonacci sequence, F_3 to F_7.

b. (3 pts) Consider the ratio of successive terms: $r_n = \frac{F_{n+1}}{F_n}$. Write the first five terms in this sequence, r_1 to r_5.

c. (9 pts) Show that $r_n = 1 + \dfrac{1}{r_{n-1}}$, for $n > 1$. Assuming that the sequence $\{r_n\}$ converges, find its exact limit.

Problem 5. (10 pts) A hawk flying at 15 m/s at an altitude of 180 m accidentally drops its prey. The parabolic trajectory of the falling prey is described by the equation

$$y = 180 - \frac{x^2}{45}$$

until it hits the ground, where y is its height above the ground and x is the horizontal distance travelled (in meters). Calculate the distance traveled by the prey from the time it is dropped until the time it hits the ground. Express your answer as an integral (which you do not need to evaluate).

Problem 6. (15 pts) Consider the parametric curve of the cycloid,

$$x(\theta) = r(\theta - \sin\theta) \text{ and } \ y(\theta) = r(1 - \cos\theta)$$

a. (3 pts) Carefully draw one arch of the cycloid. In particular, capture the vertical and horizontal slopes correctly.

b. (7 pts) Represent the length of one arch of the cycloid as an integral.

c. (5 pts) Compute the exact value of this integral.