MAT229 Test 3 (Fall 2014): Series

Name:

Directions: Problems are not equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1. (13 pts) Power series.

a. (3 pts) For any power series, there are only three distinctly different possibilities for the radius of convergence. What are they?

b. (10 pts) Find a power series representation for the following function and determine its radius and interval of convergence:

$f(x) = \ln(5 - x)$

Problem 2. (10 pts) Find the radius of convergence and interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(-2)^n (x+3)^n}{\sqrt{n}}$$

(don't forget to check boundary points).

Problem 3. (20 pts)

a. Evaluate the indefinite integral as a power series

$$\int \frac{dx}{1+x^5}$$

b. Use your answer above to approximate the definite integral below to within .001 of the true value I:

$$I = \int_0^{\frac{1}{2}} \frac{dx}{1+x^5}$$

Problem 4. (12 pts)

a. (3 pts) Given that $a_n = f(n)$. Then what properties of $f(x)$ will guarantee that $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)dx$ diverge or converge **together**?

b. (3 pts) What are the requirements for convergence when using the Alternating Series Test (AST) for

$$S = \sum_{n=1}^{\infty} (-1)^n b_n$$

c. (3 pts) Given two series with positive terms, $S = \sum_{n=1}^{\infty} a_n$ and $T = \sum_{n=1}^{\infty} b_n$.
Suppose that $a_n < b_n\ \forall n$. Describe how one could use the comparison test to demonstrate convergence of one given convergence of the other, or to demonstrate divergence of one given divergence of the other.

Problem 5. (20 pts) Determine whether the following series are divergent or convergent. If convergent, find the sum.

a. (I'm assuming that you'll use the simplest pattern you can find to continue this series, and hence answer the question:)

$$\frac{2}{3} - \frac{4}{9} + \frac{8}{27} - \frac{16}{81} + \frac{32}{243} - \cdots$$

b.

$$\sum_{n=1}^{\infty} \ln\left(\frac{n^2+1}{2n^2+1}\right)$$

Problem 6 (25 pts). For the following series, use whatever test you wish to demonstrate whether they're convergent or divergent.

a. $\sum_{n=1}^{\infty}(-1)^n \cos\left(\frac{1}{n^2}\right)$

b. $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\cosh(n)}$

c. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+1}}$

d. $\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n^2}$

e. $\sum_{n=1}^{\infty} \frac{2^{n-1}3^{n+1}}{n^n}$