

MAT115 Exam 1 (Fall 2015)

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it).

FYI: 1 1 2 3 5 8 13 21 34 55 89 144 233 377 610 987 1597. Good luck!

Problem 1: (10 pts) Counting by partition (“primitive counting”):

- a. You have 53 sheep. Illustrate how to count them by partition: what string of sheep will you report to the king?

- b. As priest for the king, your job is to translate the strings the peasants send into numbers of sheep, so that the peasants can be taxed. A peasant reports the string “1, 0, 1, 0, 1, 1, 0, 0”, meaning that their first splitting of the sheep resulted in one left over, and so on. How many sheep does the peasant have? **Show your work** (e.g. draw a tree)!

Problem 3: (20 pts) Write each of the following numbers in Babylonian and in Mayan:

Number	Babylonian	Mayan
61		
3600		
6120		
10852		

Problem 4: (10 pts) The Great Fraudini

a. (4 pts) The Great Fraudini is in front of a large crowd, delighting yet another audience with his famous card trick. He has asked an audience member to choose a number from 1 to 63. The audience member writes the number on a blackboard while Fraudini is off stage, and then the number is erased. When Fraudini returns the audience member hands him three cards (out of the six cards) with the chosen number on them: 1, 2, and 16. What number did the audience member choose?

b. (6 pts) Now, to enhance the trick, Fraudini decides to swallow the cards before “guessing” the number. An audience member chooses a number, and hands him four cards. He glances at the first card’s number, but then chokes to death on the card as he attempts to swallow it. You, as his assistant, rush onto the stage, push his body aside, and attempt to finish the trick.

If the remaining cards you take from Fraudini’s hand are the cards beginning with 8, 4, and 32, then what are **all** the numbers that the audience member could possibly have chosen?

Problem 5: (12 pts) Fibonacci Nim. You and I are playing a game of Fibonacci Nim with a given number of pieces of candy. In each of the three cases below, we start with the number of candies specified. You are to

- explain why you would rather go first or second, and
- then describe your **first** move (assuming that you were player one). If you’re in a bad situation, play a slow-down strategy.

Number	Player 1 or 2?	What would be your first move?
79		
89		
96		

Problem 6: (20 pts) Egyptian Math

a. Use Egyptian Multiplication to multiply $57 \cdot 132$.

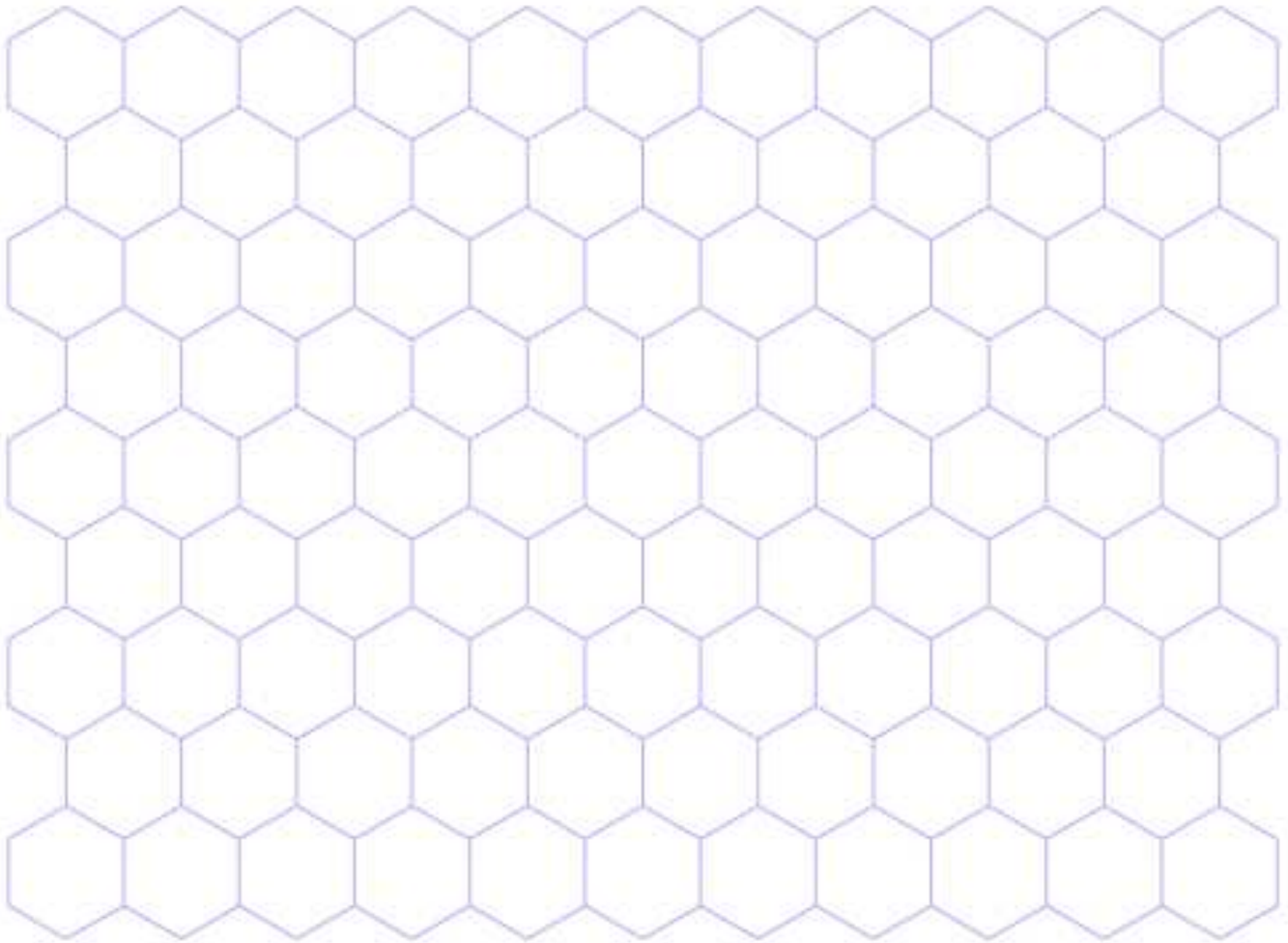
b. Divide 9 loaves among 17 people, writing the answer so as to make the ancient Egyptians happy (using only unit fractions). You may use either of our two methods (the unit table, or the doubling/halving table).

A short $2/n$ table from the Rhind Mathematical Papyrus

$2/3 = 1/2 + 1/6$	$2/5 = 1/3 + 1/15$	$2/7 = 1/4 + 1/28$
$2/9 = 1/6 + 1/18$	$2/11 = 1/6 + 1/66$	$2/13 = 1/8 + 1/52 + 1/104$
$2/15 = 1/10 + 1/30$	$2/17 = 1/12 + 1/51 + 1/68$	$2/19 = 1/12 + 1/76 + 1/114$
$2/21 = 1/14 + 1/42$	$2/23 = 1/12 + 1/276$	$2/25 = 1/15 + 1/75$
$2/27 = 1/18 + 1/54$	$2/29 = 1/24 + 1/58 + 1/174 + 1/232$	$2/31 = 1/20 + 1/124 + 1/155$
$2/33 = 1/22 + 1/66$	$2/35 = 1/30 + 1/42$	$2/37 = 1/24 + 1/111 + 1/296$
$2/39 = 1/26 + 1/78$	$2/41 = 1/24 + 1/246 + 1/328$	$2/43 = 1/42 + 1/86 + 1/129 + 1/301$
$2/45 = 1/30 + 1/90$	$2/47 = 1/30 + 1/141 + 1/470$	$2/49 = 1/28 + 1/196$
$2/51 = 1/34 + 1/102$	$2/53 = 1/30 + 1/318 + 1/795$	$2/55 = 1/30 + 1/330$

Problem 7: (18 pts)

a. (10 pts) Use the hexagonal grid to create Pascal's triangle:



b. (4 pts) What's wrong with the name "Pascal's Triangle"?

c. (4 pts) Indicate where one finds the following numbers in Pascal's Triangle, in a systematic way:

- i. the counting (natural) numbers
- ii. Triangular numbers
- iii. Powers of 2
- iv. Fibonacci numbers