# MAT329, Fall 2015: Chapter 16, plus old stuff

**Name:**

**Directions**: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

**Problem 1** Draw some contours of the function

$$f(x,y) = \frac{\sin(x^2+y^2)}{x^2+y^2}$$

Does the limit of $f$ as $(x,y) \to 0$ exist? Demonstrate!

**Problem 2** Consider the function

$$g(x,y) = xy^2 - e^y$$

Compute the gradient of $g$. Compare the mixed partials $g_{xy}$ and $g_{yx}$ - why should they be the same? What is the directional derivative of $g$ in the direction of the vector $\hat{\mathbf{i}} + \hat{\mathbf{j}}$?

**Problem 3** Compute the integral of the function $f(x, y) = \sin(x^2 + y^2)$ over the circle of radius 3 centered at the origin.

**Problem 4:** Find the center of mass of the bottom half (negative $z$ values) of the unit sphere centered at the origin. (Use symmetry!)

**Problem 5** Write the iterated integral

$$I = \int_0^1 \int_x^{2-x} dydx$$

as a sum of two iterated integrals in which the first integration is with respect to $x$. How can we compute the value of $I$ without formally integrating?

**Problem 6** Find and classify the extrema of the function $f(x, y, z) = 8x - 4z$ subject to the constraint $x^2 + 10y^2 + z^2 = 5$.

**Problem 7** Find and classify the extrema of the function $f(x, y) = xy(1 - x - y)$ on its domain.

**Problem 8** Describe and draw the object whose volume is calculated by the following integral:

$$\int_0^{\pi} \int_0^{\frac{\pi}{6}} \int_0^{2} dV$$

where $dV$ is the volume element in spherical coordinates ending in $d\rho d\phi d\theta$. Then evaluate the integral.

**Problem 9** Consider the vector function $\mathbf{F} = \langle y\ln(z) - z, x\ln(z) + 1, \frac{xy}{z} - x\rangle$.

- Demonstrate that $\mathbf{F}$ is conservative on its domain.

- What is $\nabla \times \mathbf{F}$?

- What is $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $C$ is the circle of radius 1 and center $(2, 2, 2)$ parallel to the $xy$-plane? (What if we had centered the circle at $(2, 2, 0)$?

**Problem 10** A particle starts at the point (-2,0), moves along the $x$-axis to (2,0), and then along the semicircle $y = -\sqrt{4 - x^2}$ to the starting point. Use Green's Theorem to find the work done on this particle by the force field $\mathbf{F}(x, y) = \langle x, x^3 + 3xy^2\rangle$.

**Problem 11** Consider the function

$$g(x, y) = xe^{\frac{-(x^2+y^2)}{2}}$$

a. Find and classify the critical points of $g$.

b. Compute the gradient vector at the point $P(1, 1)$.

c. Standing at the point $P(1, 1)$, you consider climbing to the highest point in the landscape. In what direction do you go, and what is the directional derivative in that direction?

d. Find the point in the domain at which the gradient has its greatest length. What is the significance of this point?

**Problem 12** Consider the vector function $\mathbf{F}(x, y) = \langle 3x^2, 2y \rangle$. Calculate the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ over the section of the parabola $y = x^2$ for $0 \leq x \leq 1$, in two ways: using the fundamental theorem of line integrals, and directly.