MAT329 Test 1: Chapter 14, Sections 1-6

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1 (28 pts). Consider the function $f(x, y) = x^2 e^{xy}$.

a. (7 pts) Compute f_x (by any method), and solve for where it equals 0.

b. (7 pts) Given that

$$\lim_{h \to 0} \frac{e^{ah} - 1}{h} = a,$$

use the definition of the partial derivative as a limit to calculate f_y.

c. (7 pts) Suppose that $x = st$ and $y = s^2 + t$. Use the chain rule and your answers above to express $\dfrac{\partial f}{\partial s}$.

d. (7 pts) Demonstrate (by computation) that f satisfies the conditions of Clairaut's theorem.

Problem 2 (32 pts). Consider

$$f(x, y) = 1 + \ln(xy - 1)$$

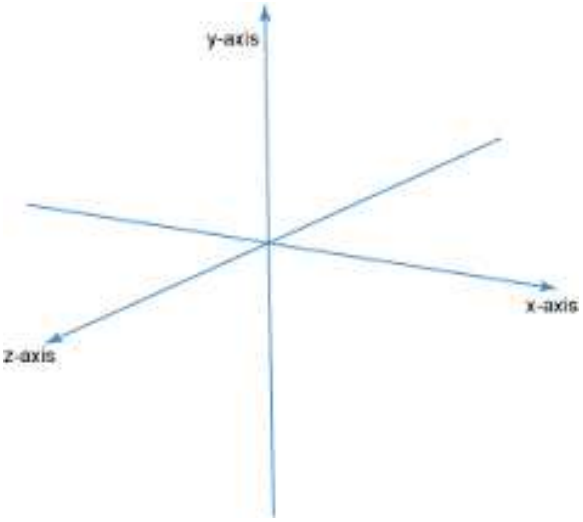
a. (3 pts) What is the domain of f?

b. (4 pts) Demonstrate (by any method) that the function f is continuous at the point $(2, 1)$.

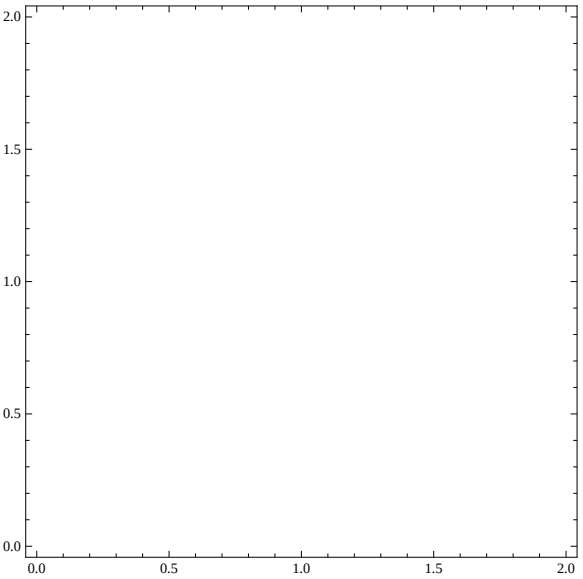
c. (8 pts) Demonstrate that the function f is differentiable at the point $(2, 1)$.

d. (5 pts) Write the linearization of f at the point $(2, 1)$. That is, write the equation of the tangent plane there.

e. (5 pts) Carefully graph that tangent plane (just the plane; not the function f).



f. (7 pts) Write an explicit equation $y = l(x)$ for level curves $f(x, y) = c$ in the xy-plane. **Carefully** draw and label a few level curves on the region $[0, 2] \times [0, 2]$. ($e \approx 2.7$)



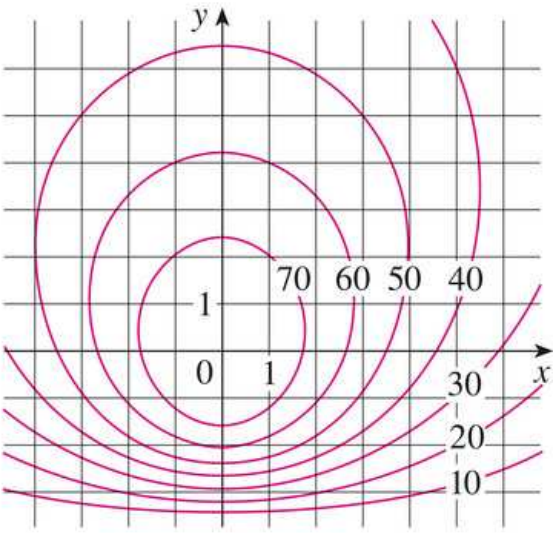
Problem 3 (20 pts). Consider the function

$$g(x, y) = \frac{x^2 + y^2}{x + y}$$

a. (10 pts) Discuss the behavior of this function at the origin. In particular, would it be continuous if we set $g(0, 0) = 0$?

b. (10 pts) What is the directional derivative of g at (1,1) in the direction of the vector $\mathbf{u} = \langle 1, 3 \rangle$?

Problem 4 (20 pts). Consider the following "terrain":



a. (10 pts) Estimate the gradient vector at the point $P(5, 1)$, and draw it on the figure. Provide rationale for both direction and length.

b. (5 pts) What are the signs (positive/negative) of the partial derivatives at the point $P(5, 1)$? Justify your answers.

f_x	f_y	f_{xx}	f_{xy}	f_{yy}

c. (5 pts) Identify the point in the region for which we would guess that the gradient has its greatest **magnitude**. How do you know?