MAT329 Test 2 (Fall 2015): Max/Mins, Lagrange, Integration

Name:

Directions: All problems are equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1. Find and classify **all** the extrema of $f(x, y) = x^2 + 3xy + y^2$ on the domain $[-1, 1] \times [-1, 1]$. What tools or tricks can you use to make your life easier?

Problem 2. Now, find the extrema of the same function, $f(x, y) = x^2 + 3xy + y^2$, subject to the constraint that (x, y) be on the unit circle (center (0,0), radius 1). Again, what tricks can you use?

Problem 3. Consider the function $f(x, y) = \sin(x)y^2$ on the rectangular region $0 \le x \le \pi$, $0 \le y \le 2$.

a. Compute the volume of the region between $z = 0$ and $z = f(x, y)$ on this region **exactly**.

b. Use two different methods to estimate the value of the integral (and compare to the exact value):

   a. The Midpoint method, on four equal-sized rectangular subregions.

   b. The "trapezoidal method" with nine points (use the corners of the same four sub-regions as for the midpoint method).

Problem 4. Consider the integral $\int_0^{1/\sqrt{2}} \int_x^{\sqrt{1-x^2}} x^2 y \mathrm{d}y \mathrm{d}x$.

a. Draw the region of integration in the plane, and compute the value of this integral.

b. Re-express the integral in polar coordinates, and evaluate to get the same answer (hopefully!).

Problem 5. Use any method to compute the volume of object shown, in the region described by $y^2 \leq x \leq a$ and $0 \leq z \leq 4 - x^2 - y^2$, and where $a = \dfrac{\sqrt{17}-1}{2}$. You can express your answer in terms of a.