MAT115 Exam 2 (Spring 2015)

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). FYI: 1 1 2 3 5 8 13 21 34 55 89 144 233 377 610 987 1597 **Good luck!**

Problem 1:   (12 pts) You and I are playing a game of Fibonacci Nim with a given number of pieces of candy. In each of the three cases below, we start with the number of candies specified. You are to explain why you would rather go first or second, and then describe your **first** move (assume that I play a slow-down strategy – but do not play out an entire game!). That is, if you go first, the first move is the only one you need to describe; if you go second, you describe my move, and then respond with the move of your own:
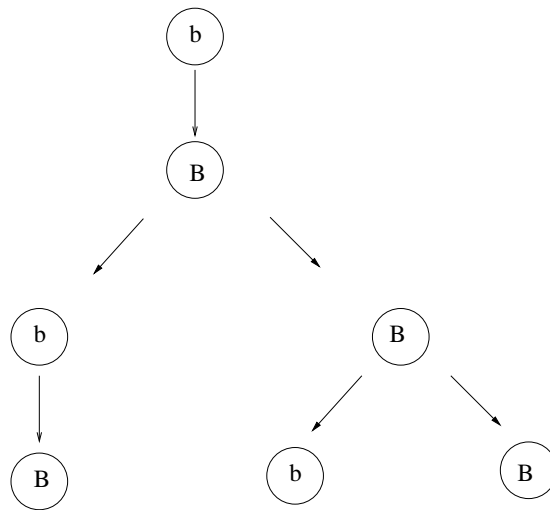
a. 79

b. 89

c. 96

Problem 2: (10 pts) Fibonacci numbers:

a. (3 pts) Describe the bunny problem that led Fibonacci to the Fibonacci numbers.

b. (4 pts) In particular, show how to continue the following graph through two more generations:

c. (3 pts) Describe how the graph above is related to the idea of fractals.

Problem 3: (10 pts) Answer four of the following five questions (write "skip" on the other):

a. The **hand-shaking theorem** is described in our reading on graphs, and says something about the degrees of the vertices in a graph. Tell me more.

b. Keith Devlin wrote a beautiful piece about Fibonacci and his life. Describe several details of Fibonacci's life that you recall from that reading.

c. Pascal's triangle contains the Fibonacci numbers in a systematic way. Show me how.

d. Tell the story of Euler and the beginnings of graph theory.

e. Draw a complete graph with 6 vertices, and describe how it relates to Facebook.

Problem 4: (10 pts) The golden ratio ϕ was obtained in class in two different ways.

a. (3 pts) What is ϕ, and what were the two ways?

b. (4 pts) In the grid provided, make the largest Fibonacci spiral you can (two copies of the grid are provided, in case you screw up!). Hint: what's the largest Fibonacci number that could be the side of a rectangle on this grid?
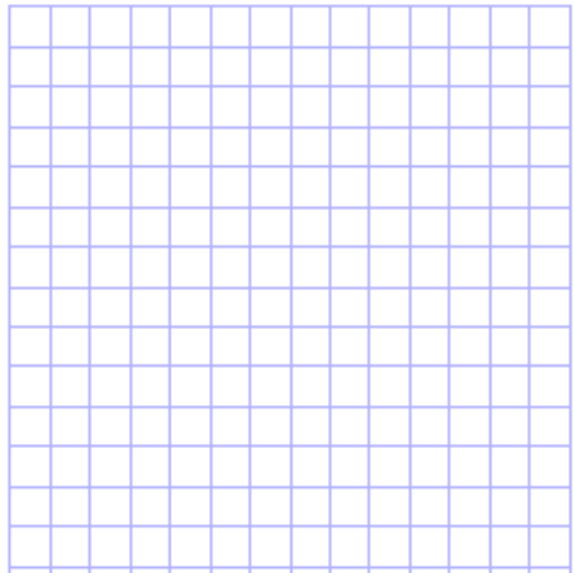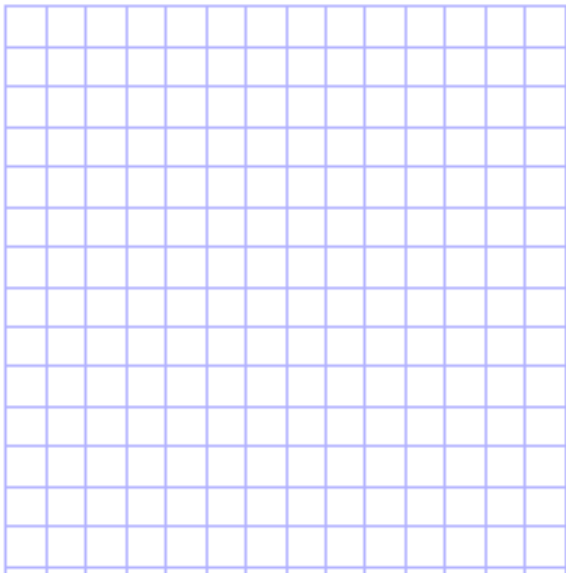
c. (3 pts) Relate this Fibonacci spiral process to
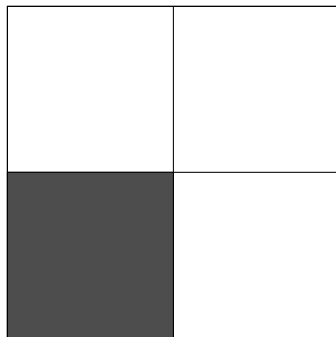
    i. the golden ratio

    ii. fractals
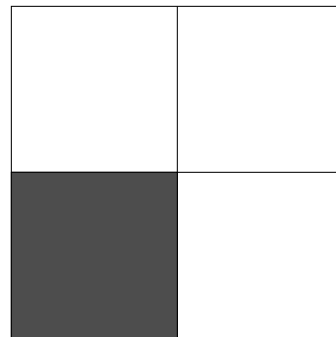
Problem 5: (10 pts) Let's start some fractals.

a. Here is the initiator and generator for an area fractal. We could describe the generator this way: "divide a square up into four equal sub-rectangles, and remove the bottom left." In the third rectangle, show what the fractal will look like after another two steps.
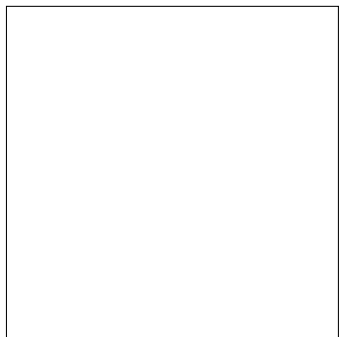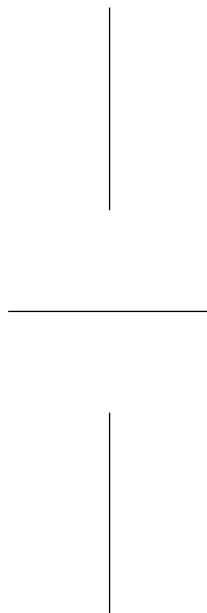
Initiator   Generator

b. Here is the initiator and generator for a stick fractal. We could describe the generator this way: "take the middle third of a stick, and rotate it 90°". To the right, show what the fractal will look like after one more step.
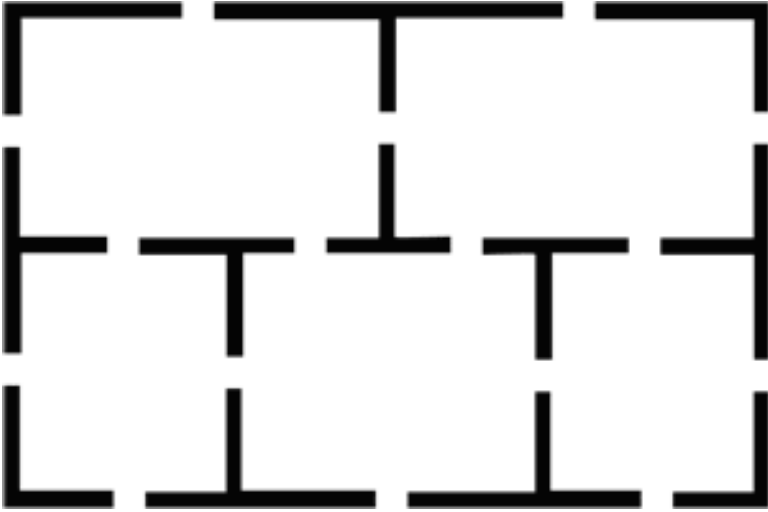
Initiator   Generator

Problem 6: (10 pts) Graphs:

a. Is it possible to start in one room, and pass through each door exactly once before going through any door twice? Use ideas from graph theory to explain.

b. Draw all simple graphs of four vertices. Which are dual to each other? (Draw dual pairs next to each other.)