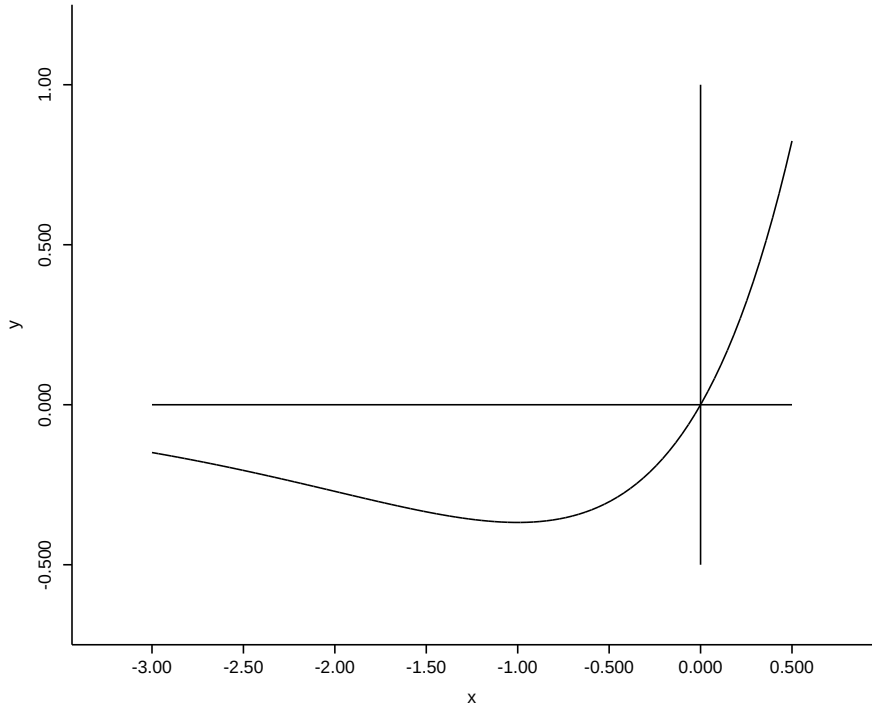
MAT360 Project 2 (Spring 2015)

Name:

Directions: While you may collaborate and discuss with others, your computer work and report should be **your own work**. You may use whatever software you wish for the calculations. Your work should be summarized in a typed report. Calculations by hand may be appended, but the summary should be nice.

Problem 1. Consider the following graph of the function $f(x) = xe^x$:



Using four different starting points for Newton's method with qualitatively different behavior for the iteration (labelling each on the graph), demonstrate the following behavior numerically:

a. A point where Newton's method converges monotonically to the root;

b. A point where Newton's method will converge non-monotonically;

c. A point where Newton's method will "blow up"; and

d. A point where Newton's method will monotonically flee the root.

Can you characterize "basins" $[a, b]$ along the real number line where each of these behaviors will occur? For example, $[a, b]$ such that $x \in [a, b]$ implies monotonic convergence.

Problem 2. Consider $f(x) = (x+2)(x+1)x(x-1)(x-2)$

a. To which root will bisection converge, if we start with the interval $[-\sqrt{2}, \pi/2]$? How many iterations must be carried out to know that?

b. Characterize intervals which contain multiple roots, yet which result in the root $x = 0$ being chosen by bisection. [You can use symmetry to make your work a little lighter.]

   You might want to define and plot a function of two variables, to give an experimental answer in some cases, and to guide your thinking.

Problem 3. Consider the following set of candidate fixed point functions $g(x)$ to solve for the root of $f(x) = e^x - \frac{1}{x^2}$. Compare them, with numerical calculations, and rank them by their convergence if we begin in the neighborhood of the root (around $r = 0.703467422$). Consider by "neighborhood" the interval $[.25, 1.5]$.

$$g_1(x) = x^3 e^x$$

$$g_2(x) = -2\ln(|x|)$$

$$g_3(x) = x\frac{x^2(x-1)e^x + 3}{x^3 e^x + 2}$$

$$g_4(x) = e^{\frac{-x}{2}}$$

Do their rates of convergence meet theoretical expectations?

Problem 4. Compare Newton's method, the Secant method, bisection, and Muller's method on Problem 3. Make a sensible choice of intervals, starting values, etc. so as to make a fair comparison. Compare rates of convergence.