Exam 3

MAT 128, Fall 2016

Show your work to receive credit

Name:
_____

1. (10 pts) A woman, 6 feet tall, walks straight away from a lampost which is 18 feet tall (from the ground to the lamp). The light is casting a shadow of her on the ground in front of her. If the tip of her shadow is moving at 8 feet per second when she is 10 feet from the post, how fast is the woman moving at that moment?

2. (15 pts) Chain rule and implicit differentiation:

    2.1. (5 pts) Illustrate the chain rule, by finding $f'(x)$, where $f(x) = \sqrt{\sin(x^2)}$.

    2.2. (10 pts) Consider the implicit equation $(y^2 - 3\,x^2) = 2xy$.

        2.2.1. Find $\frac{d}{dx}y(x)$, expressing it as a function of both x and y.

        2.2.2. Find the equation of the tangent line to the curve at the point $(1,3)$.

3. (5 pts) A rectangular tank is 4 meters high, with a square base. We aren't sure of its square base's side length, however: if the side length is known to be 5 meters plus or minus 3 cm (that is, within 3 cm of 5 meters), use differentials to approximate how the volume of the tank may vary. Find the maximum and minimum volumes actually possible, and compare to your estimates obtained using differentials.

4. (15 pts) Study the first and second derivatives of the cubic function $f(x) = 2x^3 - 3x^2 - 36x + 3$; determine

    4.1. the intervals on which f is increasing and decreasing,

    4.2. location and type of extrema (what tests can you use to determine type?),

    4.3. inflection point(s),

    4.4. and what symmetry this function possesses.

    4.5. Draw a decent sketch of $f(x)$, with the extrema and inflection points correctly located (and make sure that any other important properties of a cubic are respected).