MAT115 Exam 1 (Spring 2016)

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). Good luck!

Problem 1: (10 pts) Counting by partition ("primitive counting"):

a. You have 78 sheep. Illustrate how to count them by partition (using the tree, as in class): what string of sheep will you report to the king?

b. As priest for the king, your job is to translate the strings the peasants send into numbers of sheep, so that the peasants can be taxed. A peasant reports the string "1 0 1 0 1 1 0 0", meaning that their first splitting of the sheep resulted in none left over, and so on – so the leading 1 has a triangle around it, as I did in class. How many sheep does the peasant have? **Show your work** (e.g. draw a tree)!

Problem 2: (12 pts) Short answer:

a. Describe some **specific things** our author shares about Roman numerals.

b. What does Humphrey's counting scheme at the Furry Arms Hotel have to do with Babylonian digits?

c. What is referred to as the "five-barred gate" in our text?

d. The New York Times reported that a plant can count. Explain.

e. Describe how one-to-one correspondence relates to the idea of body counting.

f. The birthday problem: what is the smallest number of random strangers in a room that will give you better than even odds of two of them having a common birthday (so you should bet on it)?

Problem 3: (24 pts) Write each of the following numbers in Babylonian and in Mayan (show your work):

Number	Babylonian	Mayan
361		
7200		
6120		
10840		

Problem 4: (10 pts) The Great Fraudini

a. (4 pts) Explain how the Great Fraudini's trick works. How can Fraudini "read minds?" In particular, what is the mathematical fact that makes it work? It begins "Every natural number is...."

b. (6 pts) With six cards (as we used in class), each card has 32 numbers on it. If Fraudini added two new cards so that he could "read" larger numbers,

   i. what numbers would appear in the upper left-hand corners?

   ii. What would be the largest number that he could "read" (it was 63 for six cards)?

   iii. which of his eight cards would have the number 213 on them?

Problem 5: (14 pts)

a. Fibonacci Nim. You and I are playing a game of Fibonacci Nim with a given number of pieces of candy. In each of the three cases below, we start with the number of candies specified. You are to

- explain why you would rather go first or second, and
- then describe your **first** move (assuming that you were player one). If you're in a bad situation, play a slow-down strategy.

Number	Player 1 or 2?	What would be your first move?
102		
55		
37		

b. Write the five Fibonacci numbers that follow 144.

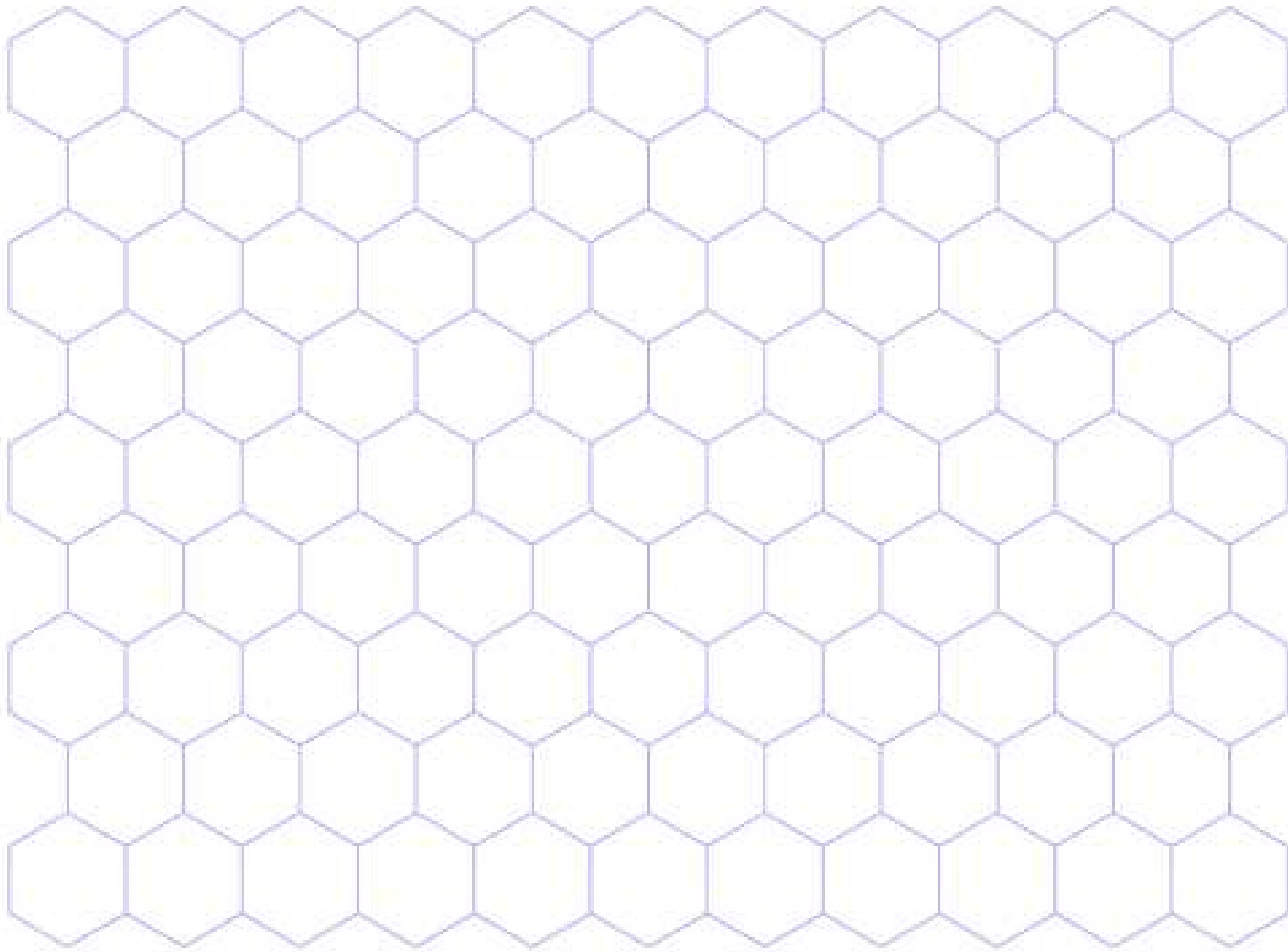
Problem 6: (10 pts)

a. (4 pts) Describe the actual problem that Fibonacci wrote about in his textbook that gave rise to the numbers that bear his name.

b. (6 pts) Suppose we change his problem so that it takes <u>two</u> months for a pair of rabbits to mature before they reproduce. Draw a tree (similar to that we used in class) to illustrate seven generations of rabbits. How many pairs would there be at the end of each month?

Problem 7: (10 pts)

a. (4 pts) Use this hexagonal grid to create Pascal's triangle, starting down from a "1" in the top row, center:



b. (3 pts) What's wrong with the name "Pascal's Triangle"?

c. (3 pts) Demonstrate how the powers of 2 appear in Pascal's triangle in a systematic way.

Problem 8: (10 pts) Translate the following Babylonian tablet, and then fill in the missing numbers: