MAT115 Exam 2 (Spring 2016)

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1: (10 pts) Eratosthenes found the primes using his sieve.

a. (5 pts) Illustrate how he did it, using this table (write all the primes to the right of the table):

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120

b. (3 pts) Describe the prime factorization theorem.

c. (2 pts) What do we **not know** about twin primes, like 17 and 19?

Problem 2: (10 pts) Symmetry:

a. I've provided space below the figure for your answers: please put them there.

All these shapes are hexagons.

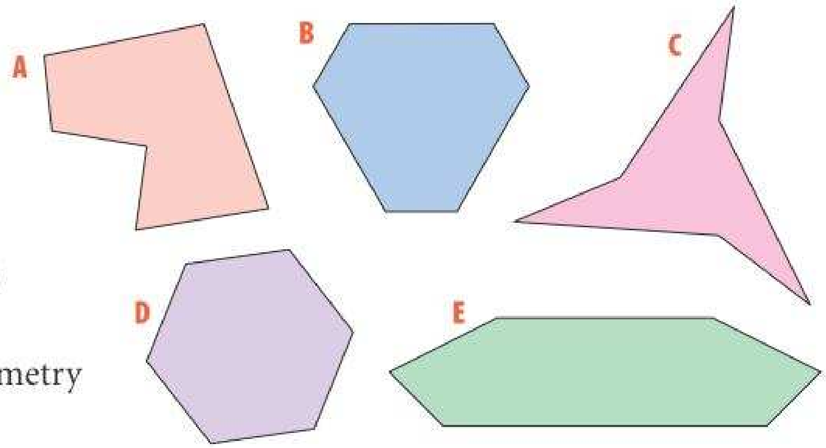
Which hexagon has

(a) only one line of symmetry

(b) rotation symmetry but no reflection symmetry

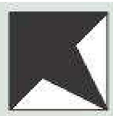
(c) rotation symmetry of order 3 and 3 lines of symmetry

(d) no reflection or rotation symmetry



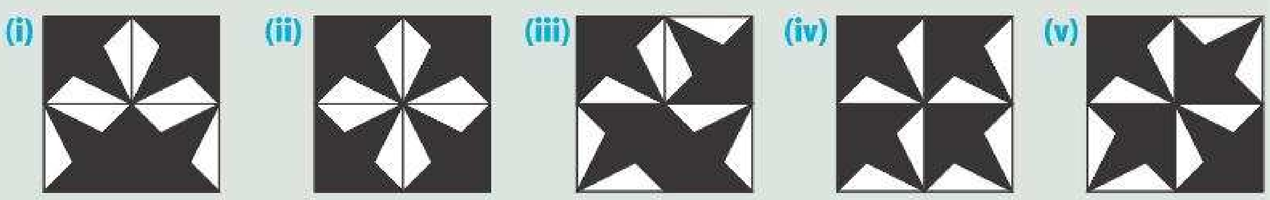
- (a)
- (b)
- (c)
- (d)

b. For Judith's problem, write your answer directly under each pattern in part (a); for (b), just fill in the proper pattern. If you mess up, draw another elsewhere on the paper.

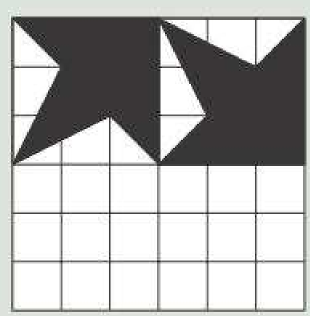
Judith has lots of tiles, all like this one. 

(a) Judith makes these patterns.

For each pattern, write down the number of lines of symmetry it has. If the pattern does not have reflection symmetry, write 0.



(b) Copy and complete this tiling pattern so that it has rotation symmetry of order 4.



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Problem 3: (10 pts) Short answer:

a. Why do we not consider 1 neither prime nor a product of primes?

b. What use did Archimedes make of regular polygons?

c. What are magic squares?

d. Harold Jacobs observes that playing cards are often symmetrical. What kind of symmetry would they have, and why would they be symmetrical?

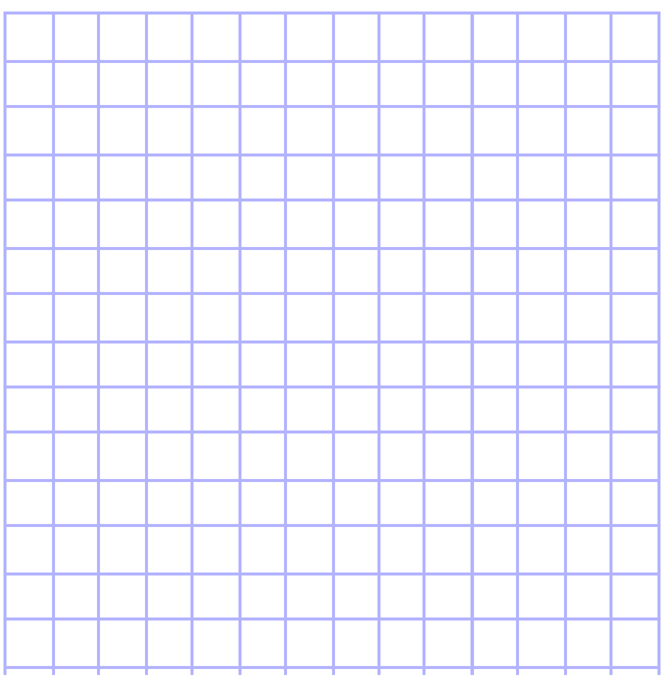
e. Tell me something that you learned about any one of Pierre de Fermat, Rene Decartes, Robert Recorde, or Gerolamo Cardano.

Problem 4: (10 pts) The golden ratio ϕ was discovered in class in two different ways.

a. (2 pts) What is the true value of ϕ? What is its four digit approximate value?

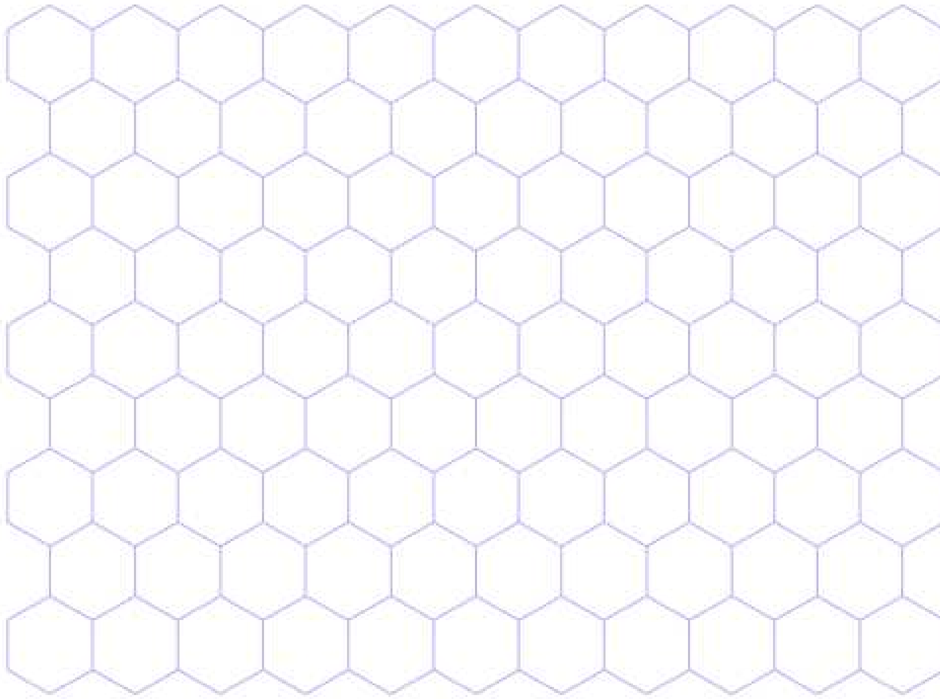
b. (4 pts) What was the Greek's definition of a golden rectangle, and how does it relate to ϕ? **Note:** you do not need to **derive** ϕ: just explain how the Greek's went about defining it (perhaps with a diagram).

c. (4 pts) In the grid provided, make the largest Fibonacci spiral you can. At right, describe the connection between this spiral process and the golden ratio and rectangle.



Problem 5: (10 pts) Pascal's triangle

a. (4 pts) Use this hexagonal grid to create Pascal's triangle, starting down from a "1" in the top row, center: Describe every system of numbers appearing in the triangle that you can think of – including



the Fibonacci numbers.

b. You have seven different cans of soda in your cooler. You reach in and pull out three at random. How many different soda combinations are possible? Justify your answer!

Problem 6: (10 pts) Demonstrate Egyptian Multiplication by multiplying

a. 27*63

b. 83*109

Problem 7: (10 pts) Demonstrate Egyptian division (give your answer as Egyptians would) for the following. You may use either of our two methods (the unit fraction table – there's a table at the end of the test – or the doubling/halving table).

a. Compute $\dfrac{23}{32}$.

b. Divide 9 loaves among 11 people.

A short 2/n table from the Rhind Mathematical Papyrus

2/3 = 1/2 + 1/6	2/5 = 1/3 + 1/15	2/7 = 1/4 + 1/28
2/9 = 1/6 + 1/18	2/11 = 1/6 + 1/66	2/13 = 1/8 + 1/52 + 1/104
2/15 = 1/10 + 1/30	2/17 = 1/12 + 1/51 + 1/68	2/19 = 1/12 + 1/76 + 1/114
2/21 = 1/14 + 1/42	2/23 = 1/12 + 1/276	2/25 = 1/15 + 1/75
2/27 = 1/18 + 1/54	2/29 = 1/24 + 1/58 + 1/174 + 1/232	2/31 = 1/20 + 1/124 + 1/155
2/33 = 1/22 + 1/66	2/35 = 1/30 + 1/42	2/37 = 1/24 + 1/111 + 1/296
2/39 = 1/26 + 1/78	2/41 = 1/24 + 1/246 + 1/328	2/43 = 1/42 + 1/86 + 1/129 + 1/301
2/45 = 1/30 + 1/90	2/47 = 1/30 + 1/141 + 1/470	2/49 = 1/28 + 1/196
2/51 = 1/34 + 1/102	2/53 = 1/30 + 1/318 + 1/795	2/55 = 1/30 + 1/330