MAT129 Exam 3 (Spring 2016)

Name:

Directions: Problems are not equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1 (20 pts). The graph of the derivative f' of a continuous function f is shown.



With justification, answer the following about the function f:

a. On which intervals is f increasing and on which is it decreasing?

b. At what values of x does f have a local extremum, and of what type?

c. On what intervals is f concave upward? Concave downward?

d. State the x-coordinate(s) of the points of inflection of f. How do you know?

e. Assuming f(0) = 0, sketch a plausible graph of f on the graph of f'(x), repeated here for your convenience:



Problem 2 (12 pts). Describe the asymptotic (end) behavior of the following functions (that is, the behavior of the function as $x \to \pm \infty$): the more detail, the better.

a.
$$f(x) = \frac{2x^2 + 7}{x^2 + 5x - 2}$$

b.
$$f(x) = \frac{x^2 - 3x + 6}{x - 6}$$

c.
$$f(x) = \sqrt{x^2 + 6x + 9}$$

Problem 3 (10 pts). Use the guidelines of our text to sketch (well!) the graph of $f(x) = \frac{x}{x^2 + 1}$. Graph the function on the axes below, incorporating all relevant information (including **labels**): the more, the better. Hint: can you make your life easier?



Problem 4 (10 pts). A farmer has 1200 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

Problem 5 (10 pts). Find f if $f''(x) = \cos(x)$, f(0) = 0, and f'(0) = 1.

Problem 6 (15 pts). Let A be the signed area of the region that lies between the graph of f(x) = 3x - 2 and the x-axis, between x = 0 and x = 2.

a. (3 pts) Draw a figure representing A.

b. (4 pts) Evaluate A using elementary geometry.

c. (4 pts) Estimate the value of A taking the sample points to be right endpoints and using four subintervals (i.e. RRR_4).

d. Problem 6, cont.: (4 pts) Using right endpoints, write an expression for A as a limit of a Riemann sum (the limit definition of the integral).

Problem 7 (10 pts). Express

$$\lim_{n \to \infty} \sum_{i=1}^{n} \left[3 + 2\sin\left(\frac{\pi i}{n}\right) \right] \frac{\pi}{n}$$

as an integral. Identify f, Δx , a, b, etc., and draw a figure illustrating this integral.