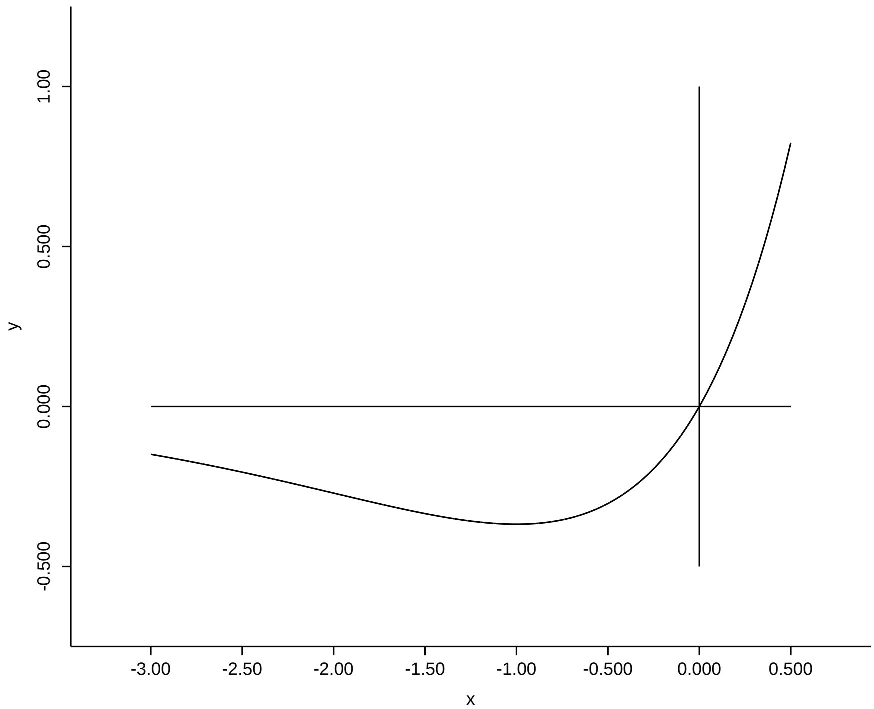
MAT360 Project 1 (Spring 2017)

Name:

Directions: While you may collaborate and discuss with others, your group's computer work and report should be **your own**. You may use whatever software you wish for the calculations. Your

work should be summarized in a single Mathematica notebook. Calculations by Mathematica or by hand may be appended, but the summary should be essentially complete.

Problem 1. Consider the following graph of the function $f(x) = xe^x$:



a. Using four different starting points for Newton's method with qualitatively different behavior for the iteration (labelling each on the graph), demonstrate the following behavior numerically:

   a. A point where Newton's method converges monotonically to the root;
   b. A point where Newton's method will converge non-monotonically;
   c. A point where Newton's method will "blow up"; and
   d. A point where Newton's method will monotonically flee the root.

b. Characterize completely the "basins" $[a, b]$ along the real number line where each of the four behaviors above will occur. For example, $[a, b]$ such that $x \in [a, b]$ implies monotonic convergence.

Problem 2. Consider $f(x) = (x+1)x(x-1)$

a. To which root will bisection converge, if we start with the interval $[-\sqrt{2}, \pi/2]$? How many iterations must be carried out to know that?

b. Characterize intervals (consider only those intervals $[a,b]$ such that $|a|, |b| \leq 4$) which

   a. contain multiple roots, yet

   b. which result in the root $x = 0$ being chosen by bisection.

   You can use symmetry to make your work a little lighter. You might want to define and plot a function of two variables ($f(a,b)$, the root chosen by $[a,b]$), to give an experimental answer in some cases, and to guide your thinking.

Problem 3. Consider the following set of candidate fixed point functions $g(x)$ to solve for the root of $f(x) = e^x - \frac{1}{x^2}$. Compare them, with numerical calculations, and rank them by their convergence if we begin in the neighborhood of the root (around $r = 0.703467422$). Consider by "neighborhood" the interval $[.25, 1.5]$.

$$g_1(x) = x^3 e^x$$

$$g_2(x) = -2\ln(|x|)$$

$$g_3(x) = x\frac{x^2(x-1)e^x + 3}{x^3 e^x + 2}$$

$$g_4(x) = e^{\frac{-x}{2}}$$

a. Verify that each is a valid fixed point function for f.

b. Do all methods converge for all values in the neighborhood?

c. How do you explain their different rates of convergence (for those which do converge)? Do their rates of convergence meet theoretical expectations?

Problem 4. Compare Newton's method, the Secant method, bisection, "quadratic Newton", and Muller's method on Problem 3. Make a sensible choice of intervals, starting values, etc. so as to make a fair comparison. Compare rates of convergence.

To generate your first guesses for secant and Muller's method, use the first few iterates from Newton's method.

If you know how to do linear regression, you can fit a model to estimate the rate of convergence:

$$\frac{|e_{k+1}|}{|e_k|^p} \approx |C|$$

so

$$\ln|e_{k+1}| \approx p\ln|e_k| + \ln|C|$$