4b p 538

Verify $x \cdot 1 = x$

$0 \cdot 1 = 0$

$1 \cdot 1 = 1$ ✓

(we'd need to verify each property, & we could do it exhaustively).

Problem arises if we have an infinite # of elements...

<u>Prove</u>: $x \cdot x = x$

(Dual to the proof top of p. 539)

$$
\begin{aligned}
x \cdot x &= x \cdot x + 0 \\
&= x \cdot x + x \cdot x' \\
&= x \cdot (x + x') \\
&= x \cdot 1 = x
\end{aligned}
$$

Proof: $x + 1 = 1$

$$x + 1 = x + (x + x') \quad 5a$$
$$= (x + x) + x' \quad 2a$$
$$= x + x' \quad \text{idempotence}$$
$$= 1 \quad 5a$$

So we've proven

$$x \cdot 0 = 0$$

by duality

4b. Prove $(x+y)' = x' \cdot y'$ (By uniqueness of complements).

Consider

$(x+y) + (x' \cdot y')$ (do we get 1?)

$= ((x+y)+x') \cdot ((x+y)+y')$ (weird dist.)

$= ((y+x)+x') \cdot (x+(y+y'))$ (commut. & assoc.)

$= (y+(x+x')) \cdot (x+(y+y'))$ (assoc.)

$= (y+1) \cdot (x+1)$ (complements)

$= 1 \cdot 1$ (J & A) proven in class

$= 1$ identity

That's half!

Consider

$$(x+y)\cdot(x'\cdot y')$$

$= ((x+y)\cdot x')\cdot y'$ assoc.

$= (x\cdot x' + y\cdot x')\cdot y'$ dist

$= (0 + y\cdot x')\cdot y'$ complements

$= (y\cdot x')\cdot y'$ identity

$= (x'\cdot y)\cdot y'$ commutativity

$= x'\cdot(y\cdot y')$ assoc.

$= x'\cdot 0$ complements

$= 0$ J & A (proven in class)

$\therefore$ by the uniqueness of complements

$$x'\cdot y' = (x+y)'$$

#9 p548

$$x \cdot y' = 0 \iff x \cdot y = x$$

(two parts to the proof:

$$x \cdot y' = 0 \implies x \cdot y = x$$

Assume $x \cdot y' = 0$. Consider

$\Rightarrow$:

$$
\begin{aligned}
x \cdot y &= x \cdot y + 0 && \text{identity} \\
&= x \cdot y + x \cdot y' && \text{(hyp)} \\
&= x \cdot (y + y') && \text{dist.} \\
&= x \cdot 1 && \text{complements} \\
&= x && \text{identity}
\end{aligned}
$$

✓

Next: $\Leftarrow$ Assume $x \cdot y = x$