$$S(n) = \sum_{i=1}^{n} i^2$$

$$\sum_{i=1}^{n} (n-i)^2 + (n+i)^2 = 2\sum_{i=1}^{n} (n^2 + i^2)$$

$$\sum_{i=0}^{n-1} i^2 + \sum_{i=n+1}^{2n} i^2 = 2(n^3 + S(n))$$

$$+ n^2 \qquad + n^2$$

$$S(2n) = \sum_{i=1}^{2n} i^2 = 2S(n) + 2n^3 + n^2$$

$$n^2(2n+1)$$

$$S(n) = 2^{\log n} \cdot 1 + \sum_{i=1}^{\log n} 2^{\log n - i}\left[(2^{i-1})^2(2^i+1)\right]$$

$$= n + \frac{n}{4}\sum_{i=1}^{\log n} 2^i(2^i+1)$$

$$= n + \frac{n}{4}\sum_{i=1}^{\log n}(4^i + 2^i)$$

$$= n + n\left(\sum_{i=1}^{\log n}\left(4^{i-1} + \frac{1}{2}2^{i-1}\right)\right)$$

$$= n + n\left(\frac{1-4^{\log n}}{1-4} + \frac{1}{2}\,\frac{1-2^{\log n}}{1-2}\right)$$

$$= n - n\left(\frac{1}{3}(1-n^2) + \frac{1}{2}(1-n)\right)$$

$$= n - \frac{n}{3} + \frac{n^3}{3} - \frac{n}{2} + \frac{n^2}{2}$$

$$= \frac{n}{6} + \frac{n^3}{2} + \frac{n^2}{3}$$

$$= \frac{n}{6}\left(1 + 3n + 2n^2\right)$$

$$\boxed{S(n) = \frac{n}{6}(2n+1)(n+1)}$$