#14 $(\forall x) P(x) \wedge (\exists x) P(x)' \rightarrow (\exists x) Q(x)$

1. $(\forall x) P(x)$ hyp
2. $(\exists x) P(x)'$ hyp
3. $((\forall x) P(x))'$ 2. negation
4. $(\exists x) Q(x)$ 1, 3, inc.

Using the 2nd principle of induction

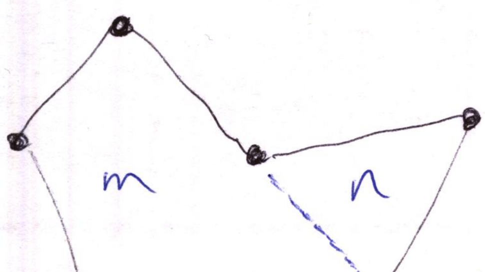
#66b: prove that the sum of the interior angles of an n-gon is $180(n-2)$, for $n \geq 3$.

1. Base case $P(n)$:

   The sum of the interior angles of a triangle is $180 = 180(3-2)$ ✓

2. By the 2nd principle of mathematical induction, assume $P(3), P(4), \ldots, P(k)$. Consider $P(k+1)$: the sum of the interior angles of a $(k+1)$-gon is $180((k+1)-2)$.

Consider a $(k+1)$-gon:




Divide it into two smaller n-gons ($n < k+1$):

$m + n = (k+1) + 2$

Sum of the interior angles of the $m$ & $n$-gons are

$180(n-2)$ (by $P(n)$) $3 \le n \le k$

$180(m-2)$ (by $P(m)$) $3 \le m \le k$

The sum of these two gives the sum of the $(k+1)$-gon:

$$180[(n-2)+(m-2)] =$$
$$180[n+m-4] =$$
$$180[(k+1)+2-4] =$$
$$180[(k+1)-2]$$

QED, by the 2nd principle : $P(n)$ $\forall n \ge 3$

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All natural numbers are interesting.

$P(n)$: $n$ is an interesting natural number

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$P(1)$: 1 is an interesting number

By the $2^{nd}$ principle: assume $P(r)$ $\forall r \in \{1, \dots, k\}$.

Consider $P(k+1)$. Then $k+1$ is interesting: if it weren't interesting then that would make it interesting!

QED.

Fibonacci #s are defined recursively:

$$\left.\begin{array}{l} F(1) = 1 \\ F(2) = 1 \end{array}\right\} \text{Base cases}$$

$F(3) = 2$

$$F(n) = F(n-1) + F(n-2)$$

Inductive step.

$F(4) \quad 3$

$F(5) = 5$

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$L(1) = 1$

$L(2) = 3$

$L(n) = L(n-1) + L(n-2) \quad n \geq 3$

$L(3) = 4$

$L(4) = 7$

$L(5) = 11$

**Prove:** $L(n) = F(n+1) + F(n-1)$ for $n \geq 2$

**Base case:** $L(2) = F(3) + F(1)$

$3 = 2 + 1$ ✓

* $L(3) = F(4) + F(2)$

$4 = 3 + 1$ ✓

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Inductive: Assume $P(k), P(k-1), \dots P(2)$.

$L(k) = F(k+1) + F(k-1)$

Consider $P(k+1)$:

$L(k+1) = F((k+1)+1) + F((k+1)-1)$

$\boxed{L(k+1) = L(k) + \underline{L(k-1)}}$ need $\frac{2^{nd}}{\text{principle}}$

$= F(k+1) + F(k-1) + F(k) + F(k-2)$

$= F(k+1) + F(k) + F(k-1) + F(k-2)$

$= F(k+2) + F(k)$

$= F((k+1)+1) + F((k+1)-1)$ ✓

Fibonaccis appear along diagonals of Pascal's triangle

sums of diagonals

1
1
2
3
5
8

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|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | 1 | | | | | |
| | | | | 1 | | 1 | | | | |
| | | | 1 | | 2 | | 1 | | | |
| | | 1 | | 3 | | 3 | | 1 | | |
| | 1 | | 4 | | 6 | | 4 | | 1 | |
| 1 | | 5 | | 10 | | 10 | | 5 | | 1 |