Consider

$$x \cdot y' = (x \cdot y) \cdot y' \quad \text{(hyp)}$$

$$= x \cdot (y \cdot y') \quad \text{assoc.}$$

$$= x \cdot 0 \quad \text{complement.}$$

$$= 0 \quad \text{by J+A (proved in class)}$$

We've proven the theorem - put it on your tool belt.

---

#12a p548

$$x + y = 0 \Rightarrow (x = 0 \land y = 0)$$

If we show that $x = 0$, then by symmetry $y = 0$. So let's show that $x = 0$. Assume $x + y = 0$

Consider

$$
\begin{aligned}
x &= x + 0 && \text{identity} \\
&= x + (x + y) && (\text{hyp}) \\
&= (x + x) + y && (\text{assoc.}) \\
&= x + y && \text{idempotence} \\
&= 0
\end{aligned}
$$

Using symmetry, $y = 0$, so

$$x + y = 0 \Rightarrow (x = 0 \wedge y = 0)$$

---

Plus we get a theorem for free!

$$\boxed{x \cdot y = 1 \rightarrow (x = 1 \wedge y = 1)}$$

Truth function for the light 3

| $s_1$ | $s_2$ | $f(s_1, s_2)$ |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 1 |

(state of the light - on or off)

$$f(s_1, s_2) = s_1 \cdot s_2 + s_1' \cdot s_2'$$

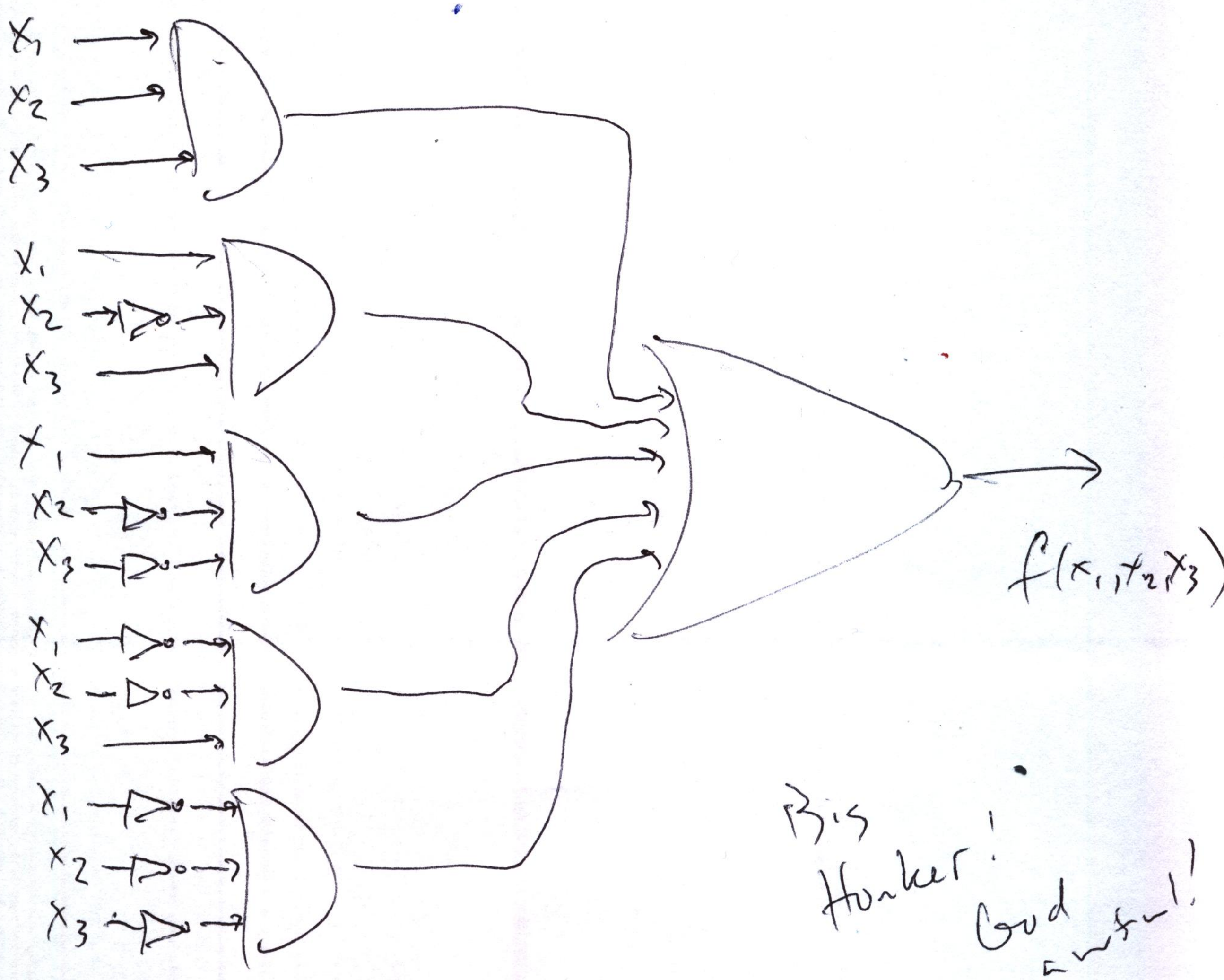
(Boolean Expression)

Practice 11, p 558

$$
\begin{aligned}
f(x_1, x_2, x_3) = {} & x_1 \cdot x_2 \cdot x_3 + \\
& x_1 \cdot x_2' \cdot x_3 + \\
& x_1 \cdot x_2' \cdot x_3' + \\
& x_1' \cdot x_2' \cdot x_3 + \\
& x_1' \cdot x_2' \cdot x_3'
\end{aligned}
$$

use dist. to collapse

collapse here, too




Big Honker! God awful!

#11

$f(x_1, x_2, x_3) =$

$x_1 \cdot x_2' \cdot x_3 +$

$x_1 \cdot x_2' \cdot x_3' +$ collapse

$x_1' \cdot x_2 \cdot x_3'$

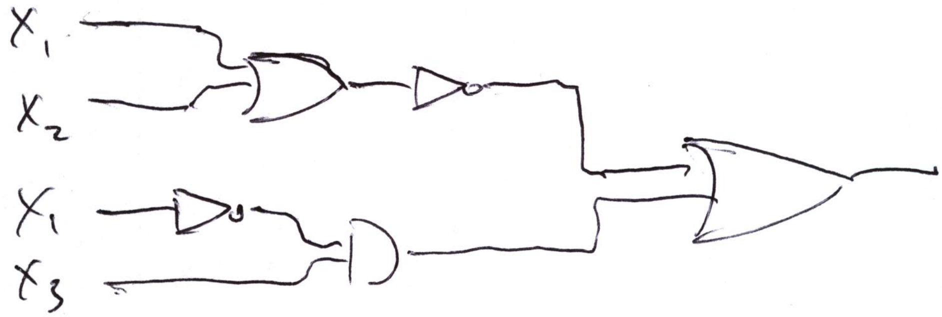
$= x_1 \cdot x_2' \cdot (x_3 + x_3') + x_1' \cdot x_2 \cdot x_3'$

$= x_1 \cdot x_2' + x_1' \cdot x_2 \cdot x_3'$

#2 p567

#1b p566

$(x_1 + x_2)' + x_1' \cdot x_3$




#2 p567

$x_1 \cdot x_2 + x_2' = f(x_1, x_2)$

| $x_1$ | $x_2$ | $f(x_1, x_2)$ |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 0 | 0 | 1 |

#2, cont.



$$(x_1' \cdot x_2)' = (x_1')' + x_2'$$

$$= x + x_2'$$

Rearm: $x_1 \cdot x_2 + x_2' = (x_1' \cdot x_2)'$

(Katiei)

$$= x + x_2'$$

Practice 11 (reprise)



Simplify:

$$x_1x_2x_3 + x_1x_2'x_3 = x_1x_3(x_2+x_2') = \underline{x_1x_3}$$

$$x_1x_2'x_3 + x_1x_2'x_3' = x_1x_2'(x_3+x_3') = \underline{x_1x_2'}$$

$$x_1'x_2'x_3 + x_1'x_2'x_3' = x_1'x_2'(x_3+x_3') = \underline{x_1'x_2'}$$

$$= x_1x_3 + x_1x_2' + x_1'x_2'$$

$$= x_1x_3 + x_2'(x_1+x_1')$$

$$= x_1x_3 + x_2'$$

---

Half Adder

Truth function(s) first:

| $x_1$ | $x_2$ | s | c |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |

$s(x_1,x_2) = x_1'x_2 + x_1x_2'$

$c(x_1,x_2) = x_1x_2$

Claim:

$$x_1' x_2 + x_1 x_2' =$$

$$(x_1 + x_2) \cdot (x_1 x_2)' =$$

$$(x_1 + x_2) \cdot (x_1' + x_2') =$$

$$x_1 x_1' + x_1 x_2' + x_2 x_1' + x_2 x_2' =$$

$$0 + x_1 x_2' + x_2 x_1' + 0 =$$

$$x_1 x_2' + x_2 x_1'$$