#35 §3.1

$\circ$	1	2	3	$\cdots$	n
1	n	n	n		n
2	n	n	n		n
3	n	n	n		n
$\vdots$					
n	n	n	n		n

$n \times n = n^2$ cells to fill.

n choices from the n elements in the set.

$n^{(n^2)}$ binary operations possible!

So $1 \circ 1$ could be $1, 2, 3, \ldots, n$

n choices.

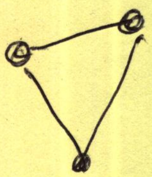
$1 \circ 2$ same

$1 \circ 3$ same

etc.

It's about "region arcs":

G



Each arc of G contributes to at most two regions

Each region must have at least 3 arcs.

"at most" "at least"

$$2a \geq 3r$$

$$r - a + n = 2$$

$$3n - 3a + 3r = 6$$

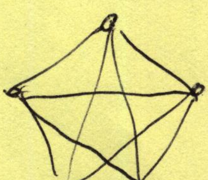
$$\leq 3n - 3a + 2a$$

$$= 3n - a$$

$$6 \leq 3n - a$$

$$a \leq 3n - 6 = 3(n-2)$$

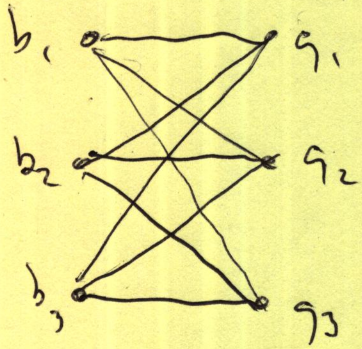
K_5: $n = 5$ $a = 10$



$\frac{n(n-1)}{2}$ arcs

$10 \leq 3 \cdot 3$ (K_5 is not planar)

To show that $K_{3,3}$ is **not** planar:



No cycles of length 3:

$2a \le 4r$

"at most" "at least"

$$r - a + n = 2$$

$$4r - 4a + 4n = 8 \le 4n - 4a + 2a$$

$$8 \le 4n - 2a$$

$$a \le 2n - 4 = 2(n-2)$$

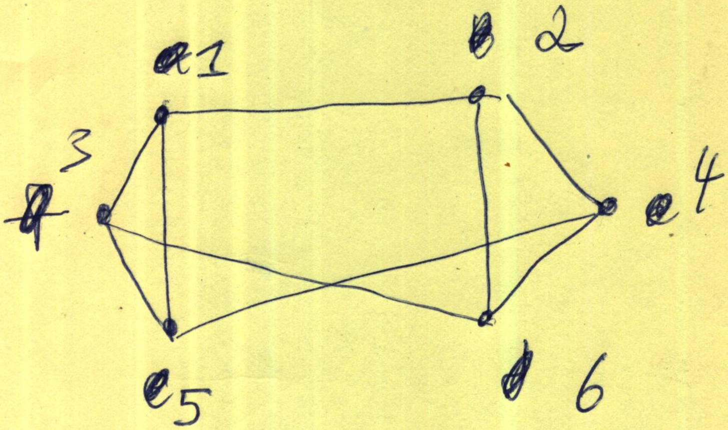
$n = 6$

$a = 9$

$9 \le 2(6-2) = 8$

So $K_{3,3}$ is **not planar**.

#26 §5.1



The easy way to show that this graph is planar is to observe that it contains neither K_5 nor $K_{3,3}$ as a subgraph – any non-planar graph will have a copy of one of those two in it. But here's an isomorphic graph to the original, showing that it is indeed planar.