Palindromic strings

Base cases:

$A = \{0,1\}$

"0" "1" $\lambda = ""$ are palindromes

Recursive step:

1. $a x a$ where a is "0" or "1" and x is a palindrome is a palindrome.

"10101"

$x = "010"$

$y = "1"$

2. $y x y$ where x & y are palindromes.

Practice 11, p 130

$T(1) = 1$

$T(n) = T(n-1) + 3 \qquad n \geq 2$

$\{1, 4, 7, 10, 13, \ldots\}$

Ultimate objective:

$$T(n) = 1 + 3(n-1)$$
$$= 3n - 2$$

<u>Closed-form solution</u>

$S(1) = a$

$S(n) = c\,S(n-1) + g(n)$

$S(2) = c\,S(1) + g(2)$

expand ... looking for patterns.

$S(3) = c\,S(2) + g(3)$

$= c\left[c\,S(1) + g(2)\right] + g(3)$

$= c^2 S(1) + c\,g(2) + g(3)$

$S(4) = c\,S(3) + g(4)$

$= c\left(c\left[c\,S(1) + g(2)\right] + g(3)\right) + g(4)$

$= c^3 S(1) + c^2 g(2) + c\,g(3) + g(4)$

⋮ guess

$S(n) = c^{n-1} S(1) + c^{n-2} g(2) + c^{n-3} g(3) + \dots$

$+ c^0 g(n)$

$P(n): S(n) = c^{n-1} S(1) + \sum_{i=2}^{n} c^{n-i} g(i)$

Proof by induction

Anchor: $S(1)$ ✓

Check $S(2)$: $S(2) = cS(1) + g(2)$

formula says:

$$S(2) = c^{2-1}S(1) + \sum_{i=2}^{2} c^{n-i} g(i)$$

$$= cS(1) + c^0 g(2)$$

$$= cS(1) + g(2) \quad ✓$$

Inductive Step:

Assume $P(k)$:

$$S(k) = c^{k-1}S(1) + \sum_{i=2}^{k} c^{k-i} g(i)$$

Consider $P(k+1)$:

$$S(k+1) = cS(k) + g(k+1)$$

$$= c\left[c^{k-1}S(1) + \sum_{i=2}^{k} c^{k-i} g(i)\right] + g(k+1)$$

$$✓ \quad = c^{(k+1)-1}S(1) + \sum_{i=2}^{k} c^{(k+1)-i} g(i) + \underbrace{g(k+1)}_{k+1^{th}\ \text{term}}$$